CompositeTransformations

Any sequence of transformations can be represented as a composite transformation matrix by calculating the product of the individual transformation matrices. Forming products of transformation matrices is usually referred to as a concatenation, or composition, of matrices.

Translations

Two **successive translations** of an object can be carried out by first concatenating the translation matrices, then applying the composite matrix to the coordinate points. Specifying the two successive translation distances as (Tx_1, Ty_1) and (Tx_2, Ty_2), we calculate the composite matrix as:

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ Tx_1 & Ty_1 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ Tx_2 & Ty_2 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ Tx_1 + Tx_2 & Ty_1 + Ty_2 & 1 \end{bmatrix}$$

whichdemonstratesthattwosuccessivetranslationsareadditive. Sotheabove equation can be written as:

$$\mathbf{T(Tx_1,Ty_1) \bullet T(Tx_2,Ty_2)=T(Tx_1+ Tx_2,Ty_1+ Ty_2)}$$

Thetransformationofcoordinatepointsforacompositetranslationis thenexpressed in matrix form as:

$$\mathbf{P' =P.T(Tx_l +Tx_2,Ty_l+ Ty_2)}$$

Scalings

Concatenatingtransformationmatricesfortwo **successivescaling**operations produces the following composite scaling matrix:

$$\mathbf{S(Sx_1,Sy_1).S(Sx_2,Sy_2)=S(Sx_1 .Sx_2,Sy_1.Sy_2)}$$

The resulting matrix in this case indicates that successive scaling operations are multiplicative. That is, if we were to triple the size of an object twice in succession,thefinalsizewouldbeninetimesthatoftheoriginal.Wecanapplytwosuccessive scaling transformationson oneobject as follows. Supposewesaymagnifytheobject2 times then magnify it 3 times. It is equivalent to saying magnify the object 6 times.

$$P' = \begin{bmatrix} sx2 & 0 & 0 \\ 0 & sy2 & 0 \\ 0 & 0 & 1 \end{bmatrix} . \begin{bmatrix} sx1 & 0 & 0 \\ 0 & sy1 & 0 \\ 0 & 0 & 1 \end{bmatrix} . P$$

$$P' = \begin{bmatrix} sx2.sx1 & 0 & 0 \\ 0 & sy2.sy1 & 0 \\ 0 & 0 & 1 \end{bmatrix} . P$$