CompositeTransformations

Rotations

The composite matrix for two successive rotations is calculatedas:

$$R(\Theta_1).R(\Theta_2)=R(\Theta_1+\Theta_2)$$

As is the case with translations, successive rotations are additive. So, rotating anobject 30 degrees anticlockwise and then rotating it again 60 degrees anticlockwise is the same as rotating the object 90 degrees anticlockwise.

TwosuccessiveRotationscanbeappliedonasingleobject.

$$P'=R1.\{R2\ .(P)\}$$

$$P'=\begin{bmatrix}\cos\theta 1 & -\sin\theta 1 & 0\\ \sin\theta 1 & \cos\theta 1 & 0\\ 0 & 0 & 1\end{bmatrix}.\begin{bmatrix}\cos\theta 2 & -\sin\theta 2 & 0\\ \sin\theta 2 & \cos\theta 2 & 0\\ 0 & 0 & 1\end{bmatrix}.P$$

$$P'=\begin{bmatrix}\cos\theta 1.\cos\theta 2-\sin\theta 2\sin\theta 1 & -\cos\theta 2\sin\theta 1-\cos\theta 1\sin\theta 2 & 0\\ \cos\theta 1.\sin\theta 2+\cos\theta 2\sin\theta 1 & -\sin\theta 1.\sin\theta 2+\cos\theta 2\cos\theta 1 & 0\\ 0 & 0 & 1\end{bmatrix}.P$$

$$P'=\begin{bmatrix}\cos(\theta 1+\theta 2) & -\sin(\theta 1+\theta 2) & 0\\ \sin(\theta 1+\theta 2) & \cos(\theta 1+\theta 2) & 0\\ 0 & 0 & 1\end{bmatrix}.P$$

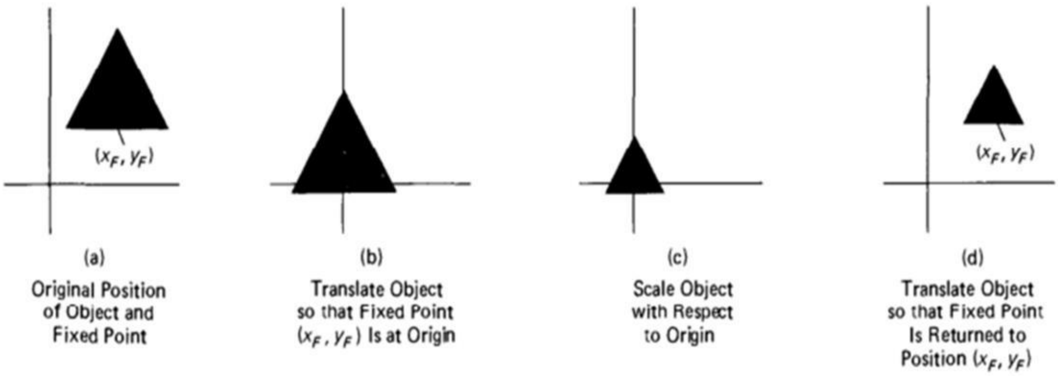
CompositeTransformationofVariousType

Wecanapplyvarioustypesoftransformationsonasingleobject.Weapply Composite transformation for Fixed Point Scaling and Pivot-Point Rotation.

ScalingRelativetoaFixedPoint

Using the transformation matrices for translation and scaling, we can obtain the composite matrix for scaling with respect to a fixed point (x_f, y_f) by considering a sequence of three transformations. This transformation sequence is illustrated in the figure in below. First, all coordinates are translated so that the fixed point is moved to the coordinate origin. Second, coordinates are scaled with respect to the origin. Third, the coordinates are translated so that the fixed point is returned to its original position. The matrix multiplications for this sequence yield:

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -x_F & -y_F & 1 \end{bmatrix} \cdot \begin{bmatrix} Sx & 0 & 0 \\ 0 & Sy & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ x_F & y_F & 1 \end{bmatrix} = \begin{bmatrix} Sx & 0 & 0 \\ 0 & Sy & 0 \\ (1 - Sx)x_F & (1 - Sy)y_F & 1 \end{bmatrix}$$



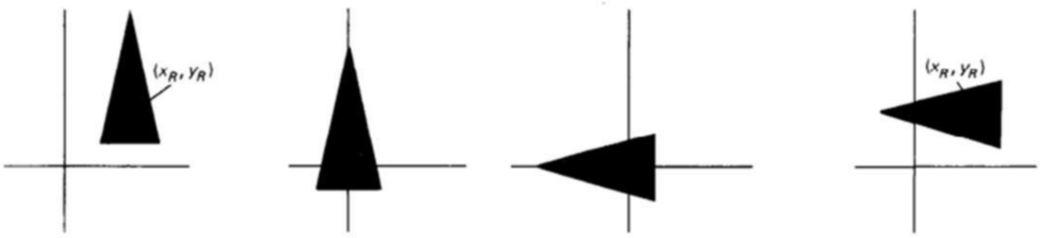
Sequence of transformations necessary to scale an object with respect to a fixed point

RotationAboutaPivotPoint

The figure in below illustrates a transformation sequence for obtaining the composite matrix for rotation about a specified pivot point (x_r, y_r). First, the object is translated so that the pivot point coincides with the coordinated origin. Second, the object is rotated about the origin. Third, the object is translated so that the pivot point returns to its original position. This sequence is represented by the matrix product:

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -x_R & -y_R & 1 \end{bmatrix} \cdot \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ x_R & y_R & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ (1-\cos\theta)x_R + y_R \cdot \sin\theta & (1-\cos\theta)y_R - x_R \cdot \sin\theta & 1 \end{bmatrix}$$



(a) Original Position of Object and Pivot Point

(b) Translation of Object so that Pivot Point (x_R, y_R) Is at Origin

(c) Rotation about Origin

(d) Translation of Object so that the Pivot Point Is Returned to Position (x_R, y_R)

Sequence of transformations necessary to rotate an object about a pivot point

Example:

There is a triangle **ABC**, **A**(0 , 0), **B**(1, 1), **C**(5, 2). Scale the image twice as large. Then translate it one unit to the left.

First of all we will make Object matrix, Scaling matrix, Translation matrix according to the values given in thequestion. Sincewe are translating theobject in left direction. So tx will be -1. While, there

Object Matrix

$$P = \begin{bmatrix} x1 & x2 & x3 \\ y1 & y2 & y3 \\ 1 & 1 & 1 \end{bmatrix} \qquad = \begin{bmatrix} 0 & 1 & 5 \\ 0 & 1 & 2 \\ 1 & 1 & 1 \end{bmatrix}$$

Scaling Matrix

$$S(sx, sy) = \begin{bmatrix} sx & 0 & 0 \\ 0 & sy & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Translation Matrix

$$T(tx, ty) = \begin{bmatrix} 1 & 0 & tx \\ 0 & 1 & ty \\ 0 & 0 & 1 \end{bmatrix} \qquad = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Nowwewillmultiplymatricesafterarrangingtheminsequence.

P'= T .{S .(P)}

$$P' = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 5 \\ 0 & 1 & 2 \\ 1 & 1 & 1 \end{bmatrix}$$

$$P' = \begin{bmatrix} 2 & 0 & -1 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 5 \\ 0 & 1 & 2 \\ 1 & 1 & 1 \end{bmatrix}$$

$$P' = \begin{bmatrix} -1 & 1 & 9 \\ 0 & 2 & 4 \\ 1 & 1 & 1 \end{bmatrix}$$

A'=(-1,0)
B'=(1,2)
C'=(9,4)

Finally,youcanseethetransformation.