

Clustering Method

Cluster techniques may be more appropriate than histogram-oriented ones in segmenting images, where each pixel has several attributes and is represented by a vector. Cluster analysis has attracted much attention since the 1960's and has been applied in many fields such as *OCR* (optical character Recognition) system, fingerprint identification, remote sensing, biological image segmentation, and so on.

K-means is a popular cluster based segmentation method where each pixel is iteratively assigned to the nearest cluster and the cluster center position is recalculated. After each iteration the cost will decrease until the cluster configuration converges at a stable state, at which point the cost is at a local minimum.

K-means clustering Algorithm

Step1: Given K =no. of class let $z_1(1), z_2(1), z_3(1) \dots z_k(1)$ at iteration one be the centers of the given classes.

Where $no.(1)$ represent iteration one

Where:

$$z_1(1) = x_1$$

$$z_2(1) = x_2$$

.....

$$z_k(1) = x_k$$

step2: at each iteration compute the distance between the input vectors and the means of the classes (z).

step3: Assign x to the set $s_i(k)$ where k is the iteration no.

so that $\|x - z_i(k)\| < \|x - z_j(k)\|$

for $j = 1, 2, \dots, k$ and $j \neq i$

$i = 1$

$$\|x - z_1(k)\| < \|x - z_2(k)\|$$

$$\|x - z_1(k)\| < \|x - z_3(k)\|$$

step4: calculate the new means

$$z_i(k+1) = \frac{1}{N_i} \sum_{j=1}^N x_j \quad x \in s_i(k), (s) \text{ set}$$

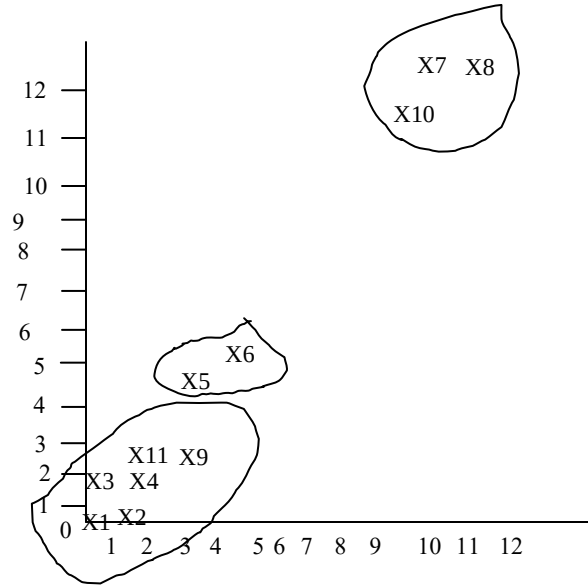
step5: check if $Z_i(k) = Z_i(k+1)$ for all $i=1,2,\dots,k$
then the algorithm converges.

Else

Repeat from **step 2**.

Example: Classify the following data set using k -means algorithm in one iteration, assume that the number of expected classes are three.

$X1=(0 \ 0)$
 $X2=(1 \ 0)$
 $X3=(0 \ 1)$
 $X4=(1 \ 1)$
 $X5=(3 \ 4)$
 $X6=(5 \ 5)$
 $X7=(10 \ 12)$
 $X8=(12 \ 12)$
 $X9=(3 \ 2)$
 $X10=(10 \ 11)$
 $X11=(2 \ 2)$



SoL. $K=3$

$$Z1(1) = (0 \ 0)$$

$$Z2(1) = (1 \ 0)$$

$$Z3(1) = (0 \ 1)$$

Compute $x4$

$$d14 = \|x4 - z1(1)\| = \sqrt{\left[\begin{pmatrix} 1 \\ 1 \end{pmatrix} - \begin{pmatrix} 0 \\ 0 \end{pmatrix}\right] \left[\begin{pmatrix} 1 \\ 1 \end{pmatrix} - \begin{pmatrix} 0 \\ 0 \end{pmatrix}\right]} = \sqrt{(1 \ 1) \begin{pmatrix} 1 \\ 1 \end{pmatrix}} = \sqrt{2} = 1.4$$

$$d24 = \|x4 - z2(1)\| = \sqrt{\left[\begin{pmatrix} 1 \\ 1 \end{pmatrix} - \begin{pmatrix} 1 \\ 0 \end{pmatrix}\right] \left[\begin{pmatrix} 1 \\ 1 \end{pmatrix} - \begin{pmatrix} 1 \\ 0 \end{pmatrix}\right]} = \sqrt{(0 \ 1) \begin{pmatrix} 0 \\ 1 \end{pmatrix}} = \sqrt{1} = 1$$

$$d34 = \|x4 - z3(1)\| = \sqrt{\left[\begin{pmatrix} 1 \\ 1 \end{pmatrix} - \begin{pmatrix} 0 \\ 1 \end{pmatrix}\right] \left[\begin{pmatrix} 1 \\ 1 \end{pmatrix} - \begin{pmatrix} 0 \\ 1 \end{pmatrix}\right]} = \sqrt{(1 \ 0) \begin{pmatrix} 1 \\ 0 \end{pmatrix}} = \sqrt{1} = 1$$

$\therefore x4 \in z2$

Compute x_5

$$d_{15} = \|x_5 - z_1(1)\| = \sqrt{\left[\begin{pmatrix} 3 \\ 4 \end{pmatrix} - \begin{pmatrix} 0 \\ 0 \end{pmatrix}\right] \left[\begin{pmatrix} 3 \\ 4 \end{pmatrix} - \begin{pmatrix} 0 \\ 0 \end{pmatrix}\right]} = \sqrt{(3 \ 4) \begin{pmatrix} 3 \\ 4 \end{pmatrix}} = \sqrt{25} = 5$$

$$d_{25} = \|x_5 - z_2(1)\| = \sqrt{\left[\begin{pmatrix} 3 \\ 4 \end{pmatrix} - \begin{pmatrix} 1 \\ 0 \end{pmatrix}\right] \left[\begin{pmatrix} 3 \\ 4 \end{pmatrix} - \begin{pmatrix} 1 \\ 0 \end{pmatrix}\right]} = \sqrt{(2 \ 4) \begin{pmatrix} 2 \\ 4 \end{pmatrix}} = \sqrt{20} = 4.4$$

$$d_{35} = \|x_5 - z_3(1)\| = \sqrt{\left[\begin{pmatrix} 3 \\ 4 \end{pmatrix} - \begin{pmatrix} 0 \\ 1 \end{pmatrix}\right] \left[\begin{pmatrix} 3 \\ 4 \end{pmatrix} - \begin{pmatrix} 0 \\ 1 \end{pmatrix}\right]} = \sqrt{(3 \ 3) \begin{pmatrix} 3 \\ 3 \end{pmatrix}} = \sqrt{18} = 4.2$$

$\therefore x_5 \in z_3$

Compute x_6

$$d_{16} = \|x_6 - z_1(1)\| = \sqrt{\left[\begin{pmatrix} 5 \\ 5 \end{pmatrix} - \begin{pmatrix} 0 \\ 0 \end{pmatrix}\right] \left[\begin{pmatrix} 5 \\ 5 \end{pmatrix} - \begin{pmatrix} 0 \\ 0 \end{pmatrix}\right]} = \sqrt{(5 \ 5) \begin{pmatrix} 5 \\ 5 \end{pmatrix}} = \sqrt{50} = 7.07$$

$$d_{26} = \|x_6 - z_2(1)\| = \sqrt{\left[\begin{pmatrix} 5 \\ 5 \end{pmatrix} - \begin{pmatrix} 1 \\ 0 \end{pmatrix}\right] \left[\begin{pmatrix} 5 \\ 5 \end{pmatrix} - \begin{pmatrix} 1 \\ 0 \end{pmatrix}\right]} = \sqrt{(4 \ 5) \begin{pmatrix} 4 \\ 5 \end{pmatrix}} = \sqrt{41} = 6.40$$

$$d_{36} = \|x_6 - z_3(1)\| = \sqrt{\left[\begin{pmatrix} 5 \\ 5 \end{pmatrix} - \begin{pmatrix} 0 \\ 1 \end{pmatrix}\right] \left[\begin{pmatrix} 5 \\ 5 \end{pmatrix} - \begin{pmatrix} 0 \\ 1 \end{pmatrix}\right]} = \sqrt{(5 \ 4) \begin{pmatrix} 5 \\ 4 \end{pmatrix}} = \sqrt{41} = 6.40$$

$\therefore x_6 \in z_2$

Compute x_7

$$d_{17} = \|x_7 - z_1(1)\| = \sqrt{\left[\begin{pmatrix} 10 \\ 12 \end{pmatrix} - \begin{pmatrix} 0 \\ 0 \end{pmatrix}\right] \left[\begin{pmatrix} 10 \\ 12 \end{pmatrix} - \begin{pmatrix} 0 \\ 0 \end{pmatrix}\right]} = \sqrt{(10 \ 12) \begin{pmatrix} 10 \\ 12 \end{pmatrix}} = \sqrt{244} = 15.62$$

$$d_{27} = \|x_7 - z_2(1)\| = \sqrt{\left[\begin{pmatrix} 10 \\ 12 \end{pmatrix} - \begin{pmatrix} 1 \\ 0 \end{pmatrix}\right] \left[\begin{pmatrix} 10 \\ 12 \end{pmatrix} - \begin{pmatrix} 1 \\ 0 \end{pmatrix}\right]} = \sqrt{(9 \ 12) \begin{pmatrix} 9 \\ 12 \end{pmatrix}} = \sqrt{225} = 15$$

$$d_{37} = \|x_7 - z_3(1)\| = \sqrt{\left[\begin{pmatrix} 10 \\ 12 \end{pmatrix} - \begin{pmatrix} 0 \\ 1 \end{pmatrix}\right] \left[\begin{pmatrix} 10 \\ 12 \end{pmatrix} - \begin{pmatrix} 0 \\ 1 \end{pmatrix}\right]} = \sqrt{(10 \ 11) \begin{pmatrix} 10 \\ 11 \end{pmatrix}} = \sqrt{221} = 14.8$$

$\therefore x_7 \in z_3$

Compute x_8

$$d_{18} = \|x_8 - z_1(1)\| = \sqrt{\left[\begin{pmatrix} 12 \\ 12 \end{pmatrix} - \begin{pmatrix} 0 \\ 0 \end{pmatrix}\right] \left[\begin{pmatrix} 12 \\ 12 \end{pmatrix} - \begin{pmatrix} 0 \\ 0 \end{pmatrix}\right]} = \sqrt{(12 \ 12) \begin{pmatrix} 12 \\ 12 \end{pmatrix}} = \sqrt{288} = 16.97$$

$$d_{28} = \|x_8 - z_2(1)\| = \sqrt{\left[\begin{pmatrix} 12 \\ 12 \end{pmatrix} - \begin{pmatrix} 1 \\ 0 \end{pmatrix}\right] \left[\begin{pmatrix} 12 \\ 12 \end{pmatrix} - \begin{pmatrix} 1 \\ 0 \end{pmatrix}\right]} = \sqrt{(11 \ 12) \begin{pmatrix} 11 \\ 12 \end{pmatrix}} = \sqrt{265} = 16.2$$

$$d_{38} = \|x_8 - z_3(1)\| = \sqrt{\left[\begin{pmatrix} 12 \\ 12 \end{pmatrix} - \begin{pmatrix} 0 \\ 1 \end{pmatrix}\right] \left[\begin{pmatrix} 12 \\ 12 \end{pmatrix} - \begin{pmatrix} 0 \\ 1 \end{pmatrix}\right]} = \sqrt{(12 \ 11) \begin{pmatrix} 12 \\ 11 \end{pmatrix}} = \sqrt{265} = 16.2$$

$\therefore x_8 \in z_2$

Compute x_9

$$d_{19} = \|x_9 - z_1(1)\| = \sqrt{\left[\begin{pmatrix} 3 \\ 2 \end{pmatrix} - \begin{pmatrix} 0 \\ 0 \end{pmatrix}\right] \left[\begin{pmatrix} 3 \\ 2 \end{pmatrix} - \begin{pmatrix} 0 \\ 0 \end{pmatrix}\right]} = \sqrt{(3 \ 2) \begin{pmatrix} 3 \\ 2 \end{pmatrix}} = \sqrt{13} = 3.6$$

$$d_{29} = \|x_9 - z_2(1)\| = \sqrt{\left[\begin{pmatrix} 3 \\ 2 \end{pmatrix} - \begin{pmatrix} 1 \\ 0 \end{pmatrix}\right] \left[\begin{pmatrix} 3 \\ 2 \end{pmatrix} - \begin{pmatrix} 1 \\ 0 \end{pmatrix}\right]} = \sqrt{(2 \ 2) \begin{pmatrix} 2 \\ 2 \end{pmatrix}} = \sqrt{8} = 2.8$$

$$d_{39} = \|x_9 - z_3(1)\| = \sqrt{\left[\begin{pmatrix} 3 \\ 2 \end{pmatrix} - \begin{pmatrix} 0 \\ 1 \end{pmatrix}\right] \left[\begin{pmatrix} 3 \\ 2 \end{pmatrix} - \begin{pmatrix} 0 \\ 1 \end{pmatrix}\right]} = \sqrt{(3 \ 1) \begin{pmatrix} 3 \\ 1 \end{pmatrix}} = \sqrt{10} = 3.1$$

$\therefore x_9 \in z_2$

Compute x_{10}

$$d_{1,10} = \|x_{10} - z_1(1)\| = \sqrt{\left[\begin{pmatrix} 10 \\ 11 \end{pmatrix} - \begin{pmatrix} 0 \\ 0 \end{pmatrix}\right] \left[\begin{pmatrix} 10 \\ 11 \end{pmatrix} - \begin{pmatrix} 0 \\ 0 \end{pmatrix}\right]} = \sqrt{(10 \ 11) \begin{pmatrix} 10 \\ 11 \end{pmatrix}} = \sqrt{221} = 14.8$$

$$d_{2,10} = \|x_{10} - z_2(1)\| = \sqrt{\left[\begin{pmatrix} 10 \\ 11 \end{pmatrix} - \begin{pmatrix} 1 \\ 0 \end{pmatrix}\right] \left[\begin{pmatrix} 10 \\ 11 \end{pmatrix} - \begin{pmatrix} 1 \\ 0 \end{pmatrix}\right]} = \sqrt{(9 \ 11) \begin{pmatrix} 9 \\ 11 \end{pmatrix}} = \sqrt{202} = 14.2$$

$$d_{3,10} = \|x_{10} - z_3(1)\| = \sqrt{\left[\begin{pmatrix} 10 \\ 11 \end{pmatrix} - \begin{pmatrix} 0 \\ 1 \end{pmatrix}\right] \left[\begin{pmatrix} 10 \\ 11 \end{pmatrix} - \begin{pmatrix} 0 \\ 1 \end{pmatrix}\right]} = \sqrt{(10 \ 10) \begin{pmatrix} 10 \\ 10 \end{pmatrix}} = \sqrt{200} = 14.1$$

$\therefore x_{10} \in z_3$

Compute x_{11}

$$d_{1,11} = \|x_{11} - z_1(1)\| = \sqrt{\left[\begin{pmatrix} 2 \\ 2 \end{pmatrix} - \begin{pmatrix} 0 \\ 0 \end{pmatrix}\right] \left[\begin{pmatrix} 2 \\ 2 \end{pmatrix} - \begin{pmatrix} 0 \\ 0 \end{pmatrix}\right]} = \sqrt{(2 \ 2) \begin{pmatrix} 2 \\ 2 \end{pmatrix}} = \sqrt{8} = 2.82$$

$$d_{2,11} = \|x_{11} - z_2(1)\| = \sqrt{\left[\begin{pmatrix} 2 \\ 2 \end{pmatrix} - \begin{pmatrix} 1 \\ 0 \end{pmatrix}\right] \left[\begin{pmatrix} 2 \\ 2 \end{pmatrix} - \begin{pmatrix} 1 \\ 0 \end{pmatrix}\right]} = \sqrt{(1 \ 2) \begin{pmatrix} 1 \\ 2 \end{pmatrix}} = \sqrt{5} = 2.23$$

$$d_{3,11} = \|x_{11} - z_3(1)\| = \sqrt{\left[\begin{pmatrix} 2 \\ 2 \end{pmatrix} - \begin{pmatrix} 0 \\ 1 \end{pmatrix}\right] \left[\begin{pmatrix} 2 \\ 2 \end{pmatrix} - \begin{pmatrix} 0 \\ 1 \end{pmatrix}\right]} = \sqrt{(2 \ 1) \begin{pmatrix} 2 \\ 1 \end{pmatrix}} = \sqrt{5} = 2.23$$

$\therefore x_{11} \in z_2$

$$S1 = \{X1\} \{(0 \ 0)\}$$

$$S2 = \{X2, X4, X6, X8, X9, X11\}$$

$$S3 = \{X3, X5, X7, X10\}$$

now compute new means

$$Z1 = X1 = (0 \ 0)$$

$$Z2 = \frac{1}{6} \begin{pmatrix} 1+1+5+12+3+2 \\ 0+1+5+12+2+2 \end{pmatrix} = \begin{pmatrix} 4 \\ 3.6 \end{pmatrix}$$

$$Z3 = \frac{1}{4} \begin{pmatrix} 0+3+10+10 \\ 1+4+12+11 \end{pmatrix} = \begin{pmatrix} 5.75 \\ 7 \end{pmatrix}$$