Exponents

We often read numbers in words such as hundred, thousands, lakhs, crores and so on. What numbers have more digits than we can read? For example, the mass of Earth is 5972190000000000000000000 kg. This cannot be read in simple words. Thus, to pronounce these types of numbers we make use of exponents. In this article, a brief introduction of exponents is given along with rules, properties and examples.

Exponent Meaning

Exponent is defined as the method of expressing large numbers in terms of powers. That means, exponent refers to how many times a number multiplied by itself. For example, 6 is multiplied by itself 4 times, i.e. $6 \times 6 \times 6 \times 6$. This can be written as 6^4 Here, 4 is the exponent and 6 is the base. This can be read as 6 is raised to power 4.

Exponent Symbol

The symbol used for representing the exponent is ^. This symbol (^) is called a carrot. For example, 4 raised to 2 can be written as 4^2 or 4^2 Thus, $4^2 = 4 \times 4 = 16$. The table below shows the representation of a few numerical expressions using exponents.

Expression	Exponent representation
$9 \times 9 \times 9 \times 9$	9^4, Base = 9, Exponent = 4

$7 \times 7 \times 7 \times 7 \times 7 \times 7 \times 7$	7^7, Base = 7, Exponent = 7
$2 \times 2 \times 2$	2^3, Base = 2, Exponent = 3

Exponent Laws

Different laws of exponents are described based on the powers they bear.

Multiplication Law: Bases – multiplying the like ones; add the exponents and keep the base the same.

When bases are raised with power to another, multiply the exponents and keep the base the same.

Division Law: Bases – dividing the like ones; subtract the exponent of the denominator from the exponent of the numerator Exponent and keep the base the same.

Let 'a' be any integer or a decimal number and 'm', 'n' are positive integers, that represent the powers to the bases such that the above laws can be written as:

- $a^m . a^n = a^{m+n}$
- $(a^m)^n = a^{mn}$
- $(ab)^n = a^n b^n$
- $(a/b)^n = a^n/b^n$
- $a^m/a^n = a^{m-n}$
- $a^m/a^n = 1/a^{n-m}$

These laws referred to the properties of exponents. These are used to simplify complex algebraic expressions and write large numbers in an understandable manner.

Exponent and Powers

As defined above, the exponent defines the number of times a number is multiplied by itself. The power is an expression that shows repeated multiplication of the same number or factor. For example, in the expression 6^4, 4 is the exponent and 6^4 is called the 6 power of 4. That means, 6 is multiplied by itself 4 times.

Exponent Formula and Rules

Exponents have certain rules which we apply in solving many problems in maths. Some of the exponent rules are given below.

Zero rule: Any number with an exponent zero is equal to 1.

Example: $8^0 = 1, a^0 = 1$

One Rule: Any number or variable that has the exponent of 1 is equal to the number or variable itself.

Example: $a^1 = a, 7^1 = 7$

Negative Exponent Rule: If the exponent value is a negative integer, then we can write the number as:

$$a^{-k} = 1/a^k$$

Example: $3^{-2} = \frac{1}{3^2} = \frac{1}{3\times3} = \frac{1}{9}$

Exponent Table

The below table shows the values of different expressions in terms of exponents along with their expansions and values. This will help you in understanding the simplification of numbers with exponents in detail.

Type of Exponent	Expression	Expansion	Simplified value
Zero exponent	6^0	1	1
One exponent	4^1	4	4
Exponent and power	2^3	$2 \times 2 \times 2$	8
Negative exponent	5^{-3}	$\frac{1}{5^3} = \frac{1}{5 \times 5 \times 5}$	$1/125$
Rational exponent	$9^{1/2}$	$\sqrt{9}$	3
Multiplication	$3^2 \times 3^3$	$3^{(2+3)} = 3^5$	273
Quotient	$7^5 / 7^3$	$7^{(5-3)} = 7^2$	49
Power of exponent	$(8^2)^2$	$8^{(2\times2)} = 8^4$	4096

Solved Examples

Example 1: Simplify $(3^2 \times 3^{-5})/\,9^{-2}$

Solution:

$$\frac{3^2 \cdot 3^{-5}}{9^{-2}}$$

$$3^2 \cdot 3^{-5} = 3^{2+(-5)} = 3^{-3}$$

$$9^{-2} = (3^2)^{-2} = 3^{-4}$$

$$\frac{3^{-3}}{3^{-4}} = 3^{-3-(-4)} = 3^{-3+4} = 3^1 = 3$$

Example 2: Simplify and write the answer in exponential form.

(i) $(2^5 \div 2^8)^5 \times 2^{-5}$

(ii) $(-4)^{-3} \times (5)^{-3} \times (-5)^{-3}$

(iii) $\left(\frac{1}{8}\right) \times (3)^{-3}$

Solution:

(i) $(2^5 \div 2^8)^5 \times 2^{-5}$

$= (2^{5-8})^5 \times 2^{-5}$

$$= (2^{-3})^5 \times 2^{-5}$$

$$= 2^{(-15-5)}$$

$$= 2^{-20}$$

Or

$$= 1/(2)^{20}$$

(ii) $(-4)^{-3} \times (5)^{-3} \times (-5)^{-3}$

$$= [(-4) \times 5 \times (-5)]^{-3}$$

$$= [100]^{-3}$$

(iii) $\left(\frac{1}{8}\right) \times (3)^{-3}$

$$= [1/(2)^3] \times (3)^{-3}$$

$$= 2^{-3} \times 3^{-3}$$

$$= (2 \times 3)^{-3}$$

$$= 6^{-3}$$

Practice Problems

1. Find m such that $(-4)^{m+1} \times (-4)^5 = (-4)^7$.
2. Find the value of $(5^0 + 4^{-1}) \times 2^2$.
3. Expand the following numbers using exponents.
   (i) 1025.63
   (ii) 1256.249

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Frequently Asked Questions on Exponent

Q1) What is an Exponent?

Exponent is the method of expressing large numbers in terms of powers. For example, 5 multiplied 3 times, i.e. $5 \times 5 \times 5$ can be expressed as $5\text{^}3$, where 3 is the exponent of 5.

Q2) How do you express the number with a rational exponent?

Consider a number with a rational exponent as $16\text{^}\{1/2\}$. This can be expressed as:

$$16^{\left\{\frac{1}{2}\right\}} = \sqrt{16} = 4$$

Q3) What is 2 with an exponent of 4?

2 with an exponent of 4 can be written as $2\text{^}4$ and its value is $2 \times 2 \times 2 \times 2 = 16$.

Q4) How do you write the negative exponent rule?

If the exponent value is a negative integer, then the number can be expressed as:

$$a\text{^}\{-k\} = 1/a\text{^}k$$

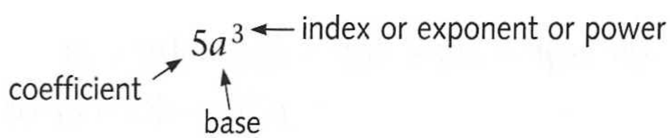
This is the negative exponent rule.

Q5) What is the zero exponent rule?

Zero exponent rule states that any number with an exponent zero (0) is equal to 1.

Powers:

The use of powers (also called indices or exponents) provides a convenient from of algebraic shorthand. Repeated factors of the same base, for example $a \times a \times a \times a$ can be written as a^4, where the number 4 indicates the number of factors multiplied together. In general, the product of n such factors a, where a and n are positive integers, is written a^n, where a is called the base and n is called the index or exponent or power. Any number multiplying a^n is called the coefficient



From the definitions above a number of rules of indices can immediately be established.

Rules of indices

1) $a^m \times a^n = a^{m+n}$ e.g. $a^5 \times a^2 = a^{5+2} = a^7$
2) $a^m \div a^n = a^{m-n}$ e.g. $a^5 \div a^2 = a^{5-2} = a^3$
3) $(a^m)^n = a^{mn}$ e.g. $(a^5)^2 = a^5 \times a^5 = a^{10}$

   these three basic rules lead to a number of important results.

4) $a^0 = 1$ because $a^m \div a^n = a^{m-n}$ and also $a^m \div a^n = \frac{a^m}{a^n}$

   then if $n = m, a^{m-m} = a^0$ and $\frac{a^m}{a^m} = 1$, so $a^0 = 1$

5) $a^{-m} = \frac{1}{a^m}$, because $a^{-m} = \frac{a^{-m} \times a^m}{a^m} = \frac{a^0}{a^m} = \frac{1}{a^m}$. So $a^{-m} = \frac{1}{a^m}$

6) $a^{\frac{1}{m}} = \sqrt[m]{a}$, because $\left(a^{\frac{1}{m}}\right)^m = a^{\frac{m}{m}} = a^1 = a$. So $a^{\frac{1}{m}} = \sqrt[m]{a}$

   from this it follows that $a^{\frac{n}{m}} = \sqrt[m]{a^n}$ or $\left(\sqrt[m]{a}\right)^n$.

Make a note of any of these results that you may be unsure about.

Example:

1- simplify $E = \left(5x^2 y^{\frac{-3}{2}} z^{\frac{1}{4}}\right)^2 \times (4x^4 y^2 z)^{\frac{-1}{2}}$

$$\frac{25x^2}{2y^4}$$

2- $F = \sqrt[3]{a^6 b^3} \div \sqrt{\frac{1}{9} a^4 b^6} \times \left(4\sqrt{a^6 b^2}\right)^{\frac{-1}{2}}$

Radicals

"Roots" (or "radicals") are the "opposite" operation of applying exponents; we can "undo" a power with a radical, and we can "undo" a radical with a power. For instance, if we square 2, we get 4, and if we "take the square root of 4", we get 2; if we square 3, we get 9, and if we "take the square root of 9", we get 3. In mathematical notation, the previous sentence means the following:

$$2^2 = 4, so\ \sqrt{4} = 2$$

$$3^2 = 9, so\ \sqrt{9} = 3$$

The " $\sqrt{..}$ " symbol used above is called the "radical" symbol. (Technically, just the "check mark" part of the symbol is the radical; the line across the top is called the "vinculum".) The expression "$\sqrt{9}$ " is read as "root nine", "radical nine", or "the square root of nine".

We can raise numbers to powers other than just 2; we can cube things (being raising things to the third power, or "to the power 3"), raise them to the fourth power (or "to the power 4"), raise them to the 100th power, and so forth. In the same way, we can take the cube root of a number, the fourth root, the 100th root, and so forth. Just as the square root undoes squaring, so also the cube root undoes cubing, the fourth root undoes raising things to the fourth power, et cetera. To indicate some root other than a square root when writing, we use the same radical symbol as for the square root, but we insert a number into the front of the radical, writing the number small and tucking it into the "check mark" part of the radical symbol. This tucked-in number corresponds to the root that you're taking. For instance, relating cubing and cube-rooting, we have:

$$4^3 = 64, so \sqrt[3]{64} = 4$$

The "3" in the radical above is called the "index" of the radical (the plural being "indices", pronounced "INN-duh-seez"); the "64" is "the argument of the radical", also called "the radicand". Perhaps because most of radicals you will see will be square roots, the index is not included on square roots. While " $\sqrt[2]{}$ " would be technically correct, I've never seen it used.

a square (second) root is written as: $\sqrt[2]{}$

a cube (third) root is written as: $\sqrt[3]{..}$

a fourth root is written as: $\sqrt[4]{..}$

a fifth root is written as: $\sqrt[5]{..}$