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Lecture 12Note!

If the system of n serially component related independent components have identical hazard then the reliability of system

$$R(t) = e^{-n \left(\frac{t}{\theta}\right)^{\beta}}$$

so the system distribution is Weibull with shape parameter β and scale parameter $\theta/n^{1/\beta}$

ex! For a system having a weibull failure distribution with shape parameter of 1.4 and scale parameter 550 days. Find the following! -

- 1- $R(100 \text{ days})$
- 2- The design life for a reliability of 0.90
- 3- median time to failure.
- 4- MTTF

Sol:

$$(1) R(100 \text{ day}) = \exp\left(-\left(\frac{100}{\theta}\right)^B\right)$$

$$R(100 \text{ day}) = \exp\left(-\left(\frac{100}{550}\right)^{1.4}\right)$$

$$(2) \therefore R(t) = e^{-\left(\frac{t}{\theta}\right)^B}$$

$$0.90 = e^{-\left(\frac{t}{550}\right)^{1.4}}$$

$$\left(-\ln 0.90\right)^{\frac{1}{1.4}} = \left(\frac{t}{550}\right)^{1.4}$$

$$\Rightarrow (-\ln 0.90) = \left(\frac{t}{550}\right)^{1.4}$$

$$t = 550 \times (-\ln 0.90)^{\frac{1}{1.4}}$$

(3) median time to failure

$$R(t) = e^{-\left(\frac{t}{\theta}\right)^B}$$

$$R(t) = e^{-\left(\frac{t}{550}\right)^{1.4}}$$

$$0.5 = e^{-\left(\frac{t}{550}\right)^{1.4}}$$

$$\left(-\ln 0.5\right)^{\frac{1}{1.4}} = \frac{t}{550}$$

$$\Rightarrow (-\ln 0.5) \times 550 = t$$

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(u) MTTF for Weibull distribution

$$MTTF = \theta \Gamma(1 + 1/\beta)$$

$$= 550 \Gamma(1 + 1/1.4)$$

ex: (H.W) ^{مثال} A power supply ^{جهاز} consist of three ^{يتكون} rectifiers ^{مستقبل} in series. Each rectifiers has a weibull distⁿ with $\beta = 2.1$. However, they have different characteristics lifetimes given by 12,000 hr, 18,500 hr and 21,500 hr. Find the MTTF

② The design life of the power supply corresponding to a reliability of 0.90.

Sol:

$$MTTF = \theta \Gamma(1 + 1/\beta)$$

∴ ^{نظر} ^{في} ^{مختلف} ^{نظم} ^{حساب} characteristic life ^{والتالي}

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$$\theta = \left[\sum_{i=1}^3 \left(\frac{1}{\theta_i} \right)^B \right]^{-1/B}$$

$$\theta = \left[\left(\frac{1}{12} \right)^{2.1} + \left(\frac{1}{18.5} \right)^{2.1} + \left(\frac{1}{21.5} \right)^{2.1} \right]^{-1/2.1}$$

$$\theta = \left[(0.0833)^{2.1} + (0.0541)^{2.1} + (0.0465)^{2.1} \right]^{-1/2.1}$$

$$\theta = (0.00921)$$

$$\Rightarrow \theta = 9.311$$

$$\therefore \text{MTTF} = 9.311 \Gamma(1 + 1/2.1)$$

$$\textcircled{2} \quad R(t) = e^{-\left(\frac{t}{\theta}\right)^B}$$

$$0.90 = \exp\left(-\frac{t}{9.311}\right)^{1/2.1}$$

$$\left(-\ln 0.90\right)^{1/2.1} = \left(t/9.311\right)^{1/2.1}$$

$$t = 9.311 \left(\ln(1/0.90) \right)^{1/2.1}$$

$$\boxed{t = 3.19}$$