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~~Lec tur~~

Lecture 3

1.5 The mean of the failure distribution is only one of several measures of central tendency of the failure distribution. Another is the median time to failure defined by

$$R(t_{med}) = 0.5 = P_r(T \geq med) \quad \cdots (8)$$

The median time to failure divides the distribution into two halves with the 50 percent occurring after median. The median time to failure may be prefered to the mean with dist^n. is skewed

1.6 A third frequently used averaged is the mode or most likely observed failure time defined by

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$$f(t_{mod}) = \max_{0 \le t < \infty} f(t) \quad \ldots\ldots \quad (9)$$

فترة

For a small fixed interval of time

تتمركز حول منوال

centered around the mode, the probability

عام أكبر

of failure with generally be greater than

موقع

for interval of the same size located

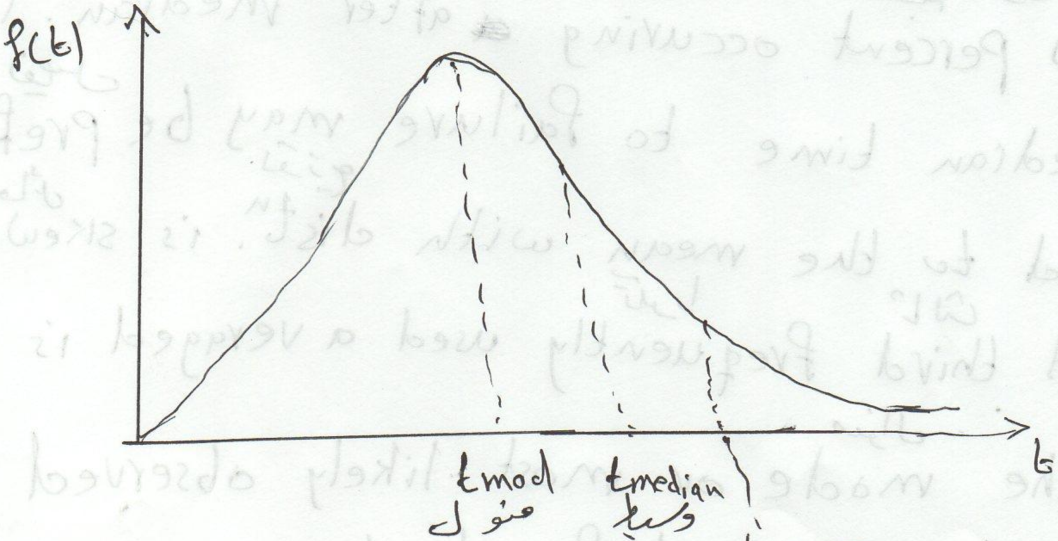
elsewhere within the distribution. The

الشكل ادناه يبين

figure below shows the locations of the

mean, the median and the mode

for a distribution skewed to the right



مقارنة المقاييس النزعة

Comparison of the measures of central

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ex! Consider the probability density function

$$f(t) = \begin{cases} 0.002\, e^{-0.002t} & t \geqslant 0 \\ 0 & \text{o.w} \end{cases}$$

الزمن مقاس بالساعات

with t in hours, Then

$$R(t) = \int_t^{\infty} 0.002\, e^{-0.002t}$$

احسب R(t)، الوسيط median time to

compute $R(t)$, median time to failure and hazard function

دالة المخاطرة

Sol!

① $$R(t) = \int_t^{\infty} 0.002\, e^{-0.002t'}\, dt'$$

$$R(t) = e^{-0.002t}$$

② $$MTTF = E(t) = \int_0^{\infty} R(t)\, dt$$

$$= \frac{-1}{0.002} \int_0^{\infty} -(0.002)\, e^{-0.002t}\, dt$$

$$MTTF = \frac{-1}{0.002}\left[e^{-0.002} \right]_{0}^{\infty}$$

$$MTTF = \frac{1}{0.002} = 500 \text{ hr}$$

To find the median

$$R(t_{med}) = e^{-0.002 t_{med}} = 0.5$$

ناخذ ln للطرفين لنحل t_{med} ماذا يساوي

$$-0.002 t_{med} = \ln 0.5$$

$$t_{med} = \frac{\ln 0.5}{-0.002} = 346.6 \text{ hr}$$

③ $\lambda(t) = \frac{f(t)}{R(t)}$ دالة المخاطرة

$$= \frac{0.002\, \cancel{e^{-0.002t}}}{\cancel{e^{-0.002t}}}$$

$$\lambda(t) = 0.002$$

لحظي آني

Note! The $\lambda(t)$ is known as instantous

hazard rate or failure rate function.

يجهز

The failure rate function $\lambda(t)$ provide

وصف

حالة بديلة

an alternative way of describing a failure

معدلات الفشل بعض الحالات

distribution. Failure rates in some cases may

متزايد

يوصف

be characterized as increasing (IFR),

ثابت

متناقص

or decreasing (dfR) or constant failure

rate (CFR)

العلاقة بين معدل المخاطرة والمعولية هي كالتالي

A particular hazard rate function will

دالة

المعولية

uniquely determine a reliability function

To see this

$$\lambda(t) = \frac{f(t)}{R(t)}$$

$$\lambda(t) = \frac{-dR(t)}{dt} * \frac{1}{R(t)}$$

$\Longrightarrow$

$$\lambda(t)\,.dt = -\frac{dR(t)}{R(t)}$$

تكامل الطرفين

$$\int_0^t \lambda(t')\,dt' = \int_1^{R(t)} \frac{-dR(t)}{R(t)}$$

where $R(0)=1$ establishes, the lower limit

جانب الايمن

تكامل

in the integral on the right-hand side

then

$$\Longrightarrow \int_0^t \lambda(t')dt' = -\ln R(t) \Big|_1^{R(t)} \quad * (-1)$$

مشتقة المقام موجود في البسط ولهذا فان تشتق التكامل هو ln المقام

$$-\int_0^t \lambda(t')d\,t' = \ln R(t) - ①$$

where $R(0)=1$, $\ln 1 = 0$

بأخذ exp للطرفين

$$\boxed{R(t) = \exp\left(-\int_0^t \lambda(t')\,dt'\right)} - (11)$$

بهذه العلاقة يتم حساب المعولية اذا علم دالة المخاطرة $\lambda(t)$

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ex: Given the linear hazard rate function

$\lambda(t) = 5 * 10^{-6} t$ where t is measured in operating hours, What is the ~~the~~ reliability function and if $R(t) = 0.98$ compute design life

اذا علمت ان دالة المخاطرة $\lambda(t)$ حيث ان t مقاس بساعات العمل ، احسب اولا الموثوقية ② اذا كانت $R(t) = 0.98$ جد نمط ~~الضا~~ الحياة t_R ؟

sol:

$$R(t) = \exp\left(-\int_0^t \lambda(t')\,dt'\right)$$

$$R(t) = \exp\left(-\int_0^t 5 \times 10^{-6}\, t'\,dt'\right)$$

$$R(t) = \exp\left(\frac{-\cancel{5}^{\,2.5} \times 10^{-6}\, t'^2}{\cancel{2}_{\,1}}\Bigg|_0^t\right)$$

$$R(t) = \exp\left(-2.5 * 10^{-6} t^2 - 0\right)$$

$$R(t) = \exp\left(-2.5 * 10^{-6} t^2\right)$$

If $R(t) = 0.98$ To find t_R

$\because R(t) = \exp(-2.5 * 10^{-6} t^2)$

$0.98 = \exp(-2.5 * 10^{-6} t^2)$

نأخذ ln للطرفين

$\ln 0.98 = -2.5 * 10^{-6} t^2$

$t^2 = \dfrac{\ln 0.98}{-2.5 * 10^{-6}}$

$t = \sqrt{\dfrac{-\ln 0.98}{2.5 * 10^{-6}}} \approx 89.89 \approx 90 \text{ hr}$