4. Hill Cipher

Hill cipher is a polygraphic substitution cipher based on linear algebra.Each letter is represented by a number modulo 26. Often the simple scheme A = 0, B = 1, …, Z = 25 is used, but this is not an essential feature of the cipher. To encrypt a message, each block of n letters (considered as an n-component vector) is multiplied by an invertible n × n matrix, against modulus 26. To decrypt the message, each block is multiplied by the inverse of the matrix used for encryption.
The matrix used for encryption is the cipher key, and it should be chosen randomly from the set of invertible n × n matrices (modulo 26).
Examples:

```
Input  : Plaintext: ACT
         Key: GYBNQKURP
Output : Ciphertext: POH
```

```
Input  : Plaintext: GFG
         Key: HILLMAGIC
Output : Ciphertext: SWK
```

Encryption

We have to encrypt the message ‘ACT’ (n=3).The key is ‘GYBNQKURP’ which can be written as the nxn matrix:

$$\begin{bmatrix} 6 & 24 & 1 \\ 13 & 16 & 10 \\ 20 & 17 & 15 \end{bmatrix}$$

The message ‘ACT’ is written as vector:

$$\begin{bmatrix} 0 \\ 2 \\ 19 \end{bmatrix}$$

The enciphered vector is given as:

$$\begin{bmatrix} 6 & 24 & 1 \\ 13 & 16 & 10 \\ 20 & 17 & 15 \end{bmatrix} \begin{bmatrix} 0 \\ 2 \\ 19 \end{bmatrix} = \begin{bmatrix} 67 \\ 222 \\ 319 \end{bmatrix} \equiv \begin{bmatrix} 15 \\ 14 \\ 7 \end{bmatrix} \pmod{26}$$

which corresponds to ciphertext of 'POH'

Decryption

To decrypt the message, we turn the ciphertext back into a vector, then simply multiply by the inverse matrix of the key matrix (IFKVIVVMI in letters).The inverse of the matrix used in the previous example is:

$$\begin{bmatrix} 6 & 24 & 1 \\ 13 & 16 & 10 \\ 20 & 17 & 15 \end{bmatrix}^{-1} \equiv \begin{bmatrix} 8 & 5 & 10 \\ 21 & 8 & 21 \\ 21 & 12 & 8 \end{bmatrix} \pmod{26}$$

For the previous Ciphertext 'POH':

$$\begin{bmatrix} 8 & 5 & 10 \\ 21 & 8 & 21 \\ 21 & 12 & 8 \end{bmatrix} \begin{bmatrix} 15 \\ 14 \\ 7 \end{bmatrix} \equiv \begin{bmatrix} 260 \\ 574 \\ 539 \end{bmatrix} \equiv \begin{bmatrix} 0 \\ 2 \\ 19 \end{bmatrix} \pmod{26}$$

which gives us back ‘ACT’.
Assume that all the alphabets are in upper case.

Example :
Consider the plaintext "pay more money" then use the encryption key

$$K = \begin{pmatrix} 17 & 17 & 5 \\ 21 & 18 & 21 \\ 2 & 2 & 19 \end{pmatrix}$$

Solution:
Take m = 3, i.e. three letters at a time. If the first **three** letters of plaintext are represented by the vector (**15 0 24)** instead of **“pay”, then**

$$K(15\ 0\ 24) = (375 \quad 819 \quad 486) \bmod 26$$
$$= (11 \quad 13 \quad 18) = LNS.$$

Decryption requires using the inverse of the matrix **K**. The inverse K^{-1} of the matrix K is defined by the equation $\mathbf{KK^{-1} = K^{-1}\ K = I}$, Where I is the identity Matrix.

$$K^{-1} = \begin{pmatrix} 4 & 9 & 15 \\ 15 & 17 & 6 \\ 24 & 0 & 17 \end{pmatrix}$$

This is demonstrated as follow:

$$\begin{pmatrix} 17 & 17 & 5 \\ 21 & 18 & 21 \\ 2 & 2 & 19 \end{pmatrix} \begin{pmatrix} 4 & 9 & 15 \\ 15 & 17 & 6 \\ 24 & 0 & 17 \end{pmatrix} = \begin{pmatrix} 443 & 442 & 442 \\ 858 & 495 & 780 \\ 494 & 52 & 365 \end{pmatrix} \bmod 26$$

$$I = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

We have, $\mathbf{C = E_K(P) = K\,P \bmod 26}$ **and** $\mathbf{P = D_K(C) = K^{-1}\,C \bmod 26}$

5. Polyalphabet Cipher:

Multi-alphabet sets are used with:

1- A set of related mono-alphabetic substitution rules is used.
2- A key determines which rule is chosen.

One famous cipher is **Vigenere** Cipher

Vigenere Cipher: Vigenere Cipher consists of 25 Caesars Cipher shifts from 0 to 25. Each is denoted by a key letter. A table is then constructed as shown in table 2-3.

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
A	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
B	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A
C	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B
D	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C
E	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D
F	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E
G	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F
H	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G
I	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H
J	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I
K	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J
L	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K
M	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L
N	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M
O	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N
P	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
Q	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P
R	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q
S	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R
T	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S
U	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T
V	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U
W	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V
X	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W
Y	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X
Z	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y

The rule of substitution:

Given a key letter X for the alphabet set and a plaintext letter **Y** for the column, i.e. in this case it is V.

Example: Encrypt the message M, where

M: "we are discovered save yourself" Using Vigenere cipher with a **key: "deceptive".**

Solution

Plaintext: **"wearediscoveredsaveyoursef"**
Key: **"deceptivedeceptivedeceptive"**

Ciphertext: "zicvtwqngrzgvtwavzhcqyglmgj"

Decryption is equally simple:

1- Key letter identifies the row.
2- Ciphertext letter in the row identifies the column.
3- Plaintext letter is at the top of the column.