Logical Design

Lecture 10

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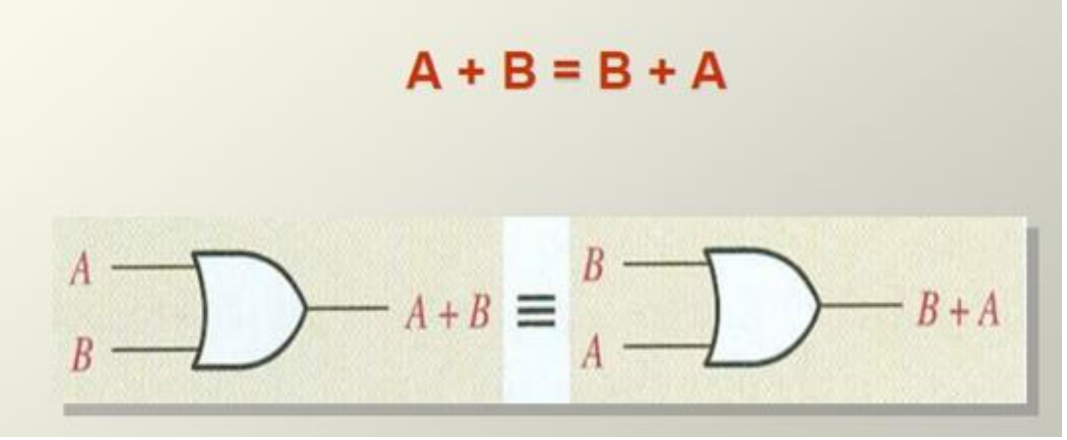
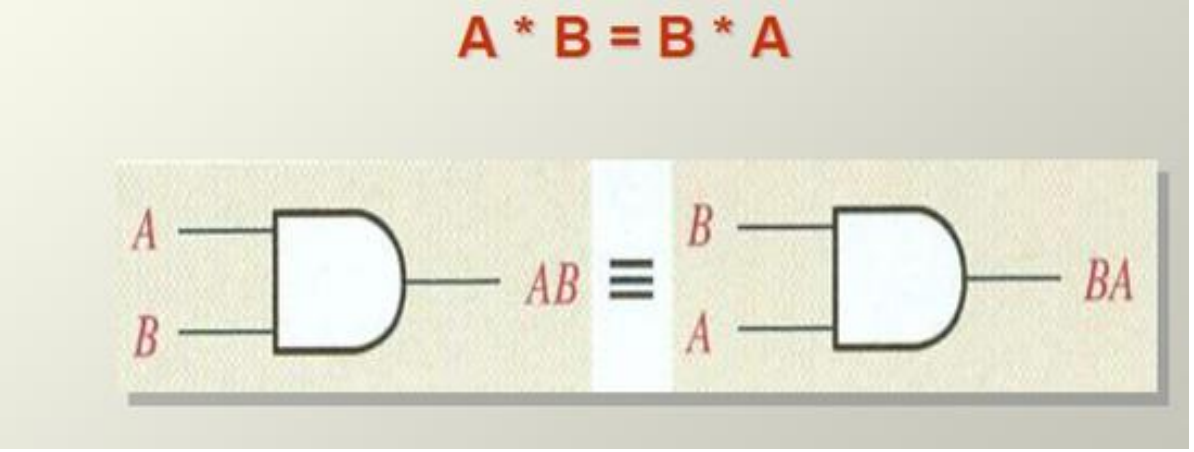
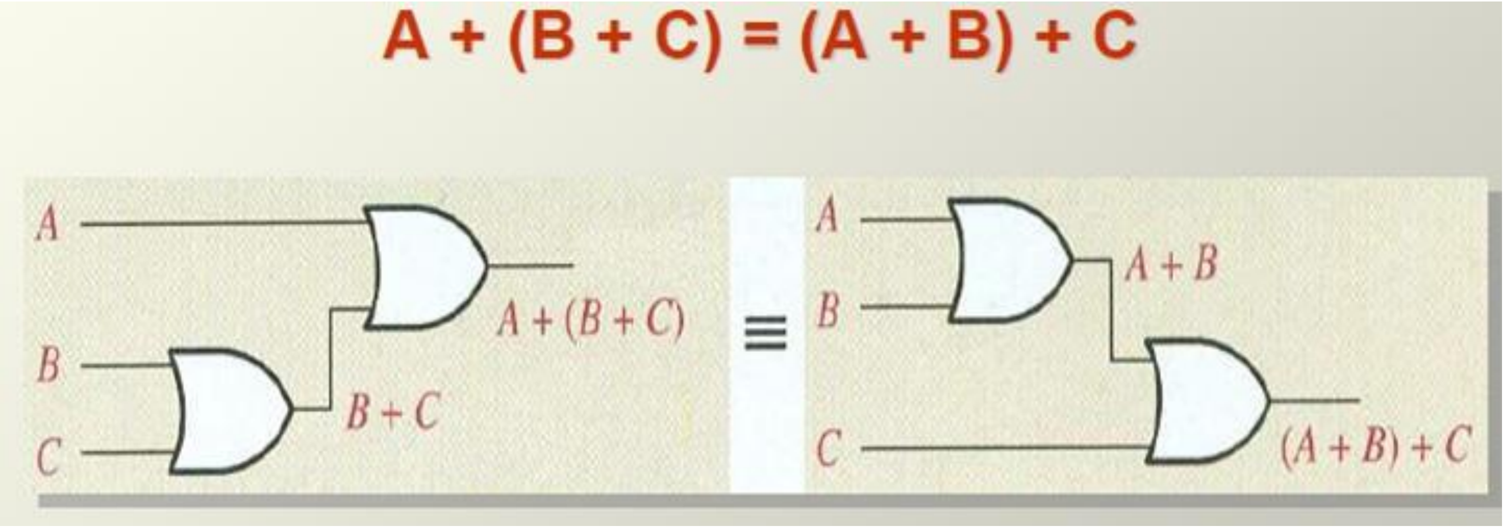
Laws of Boolean Algebra

Commutative Law

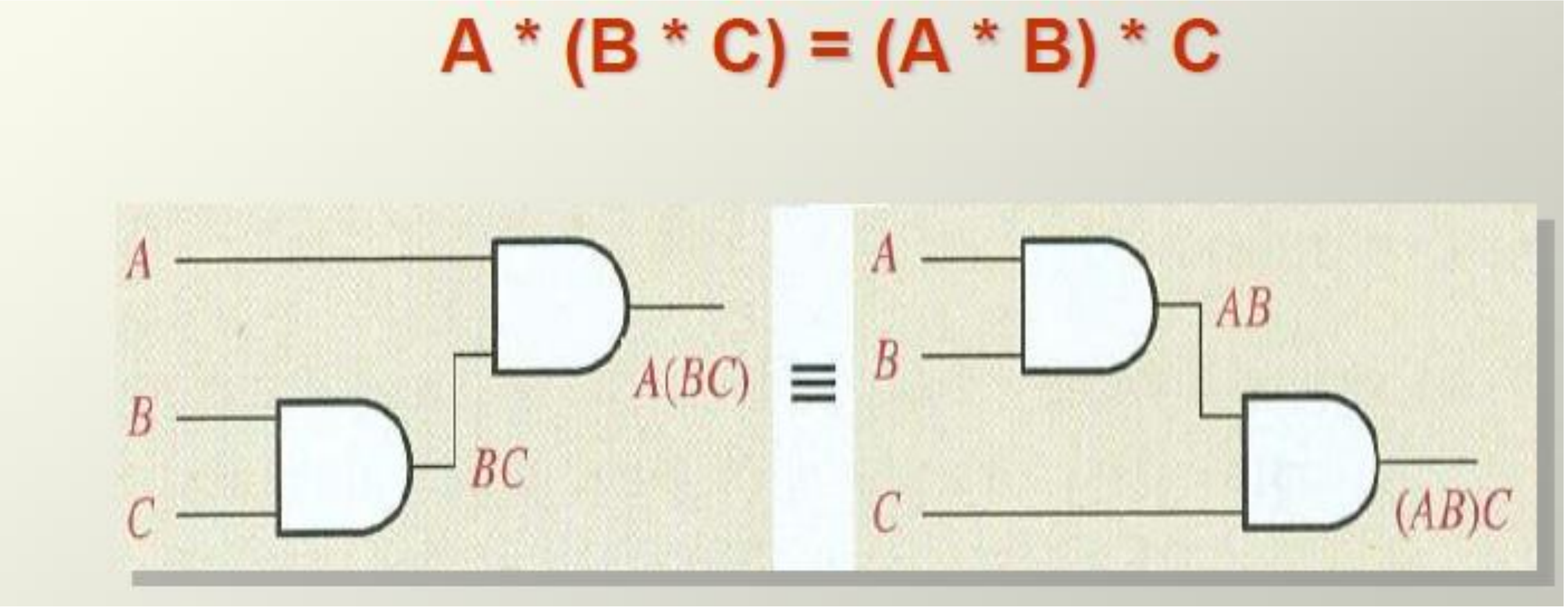




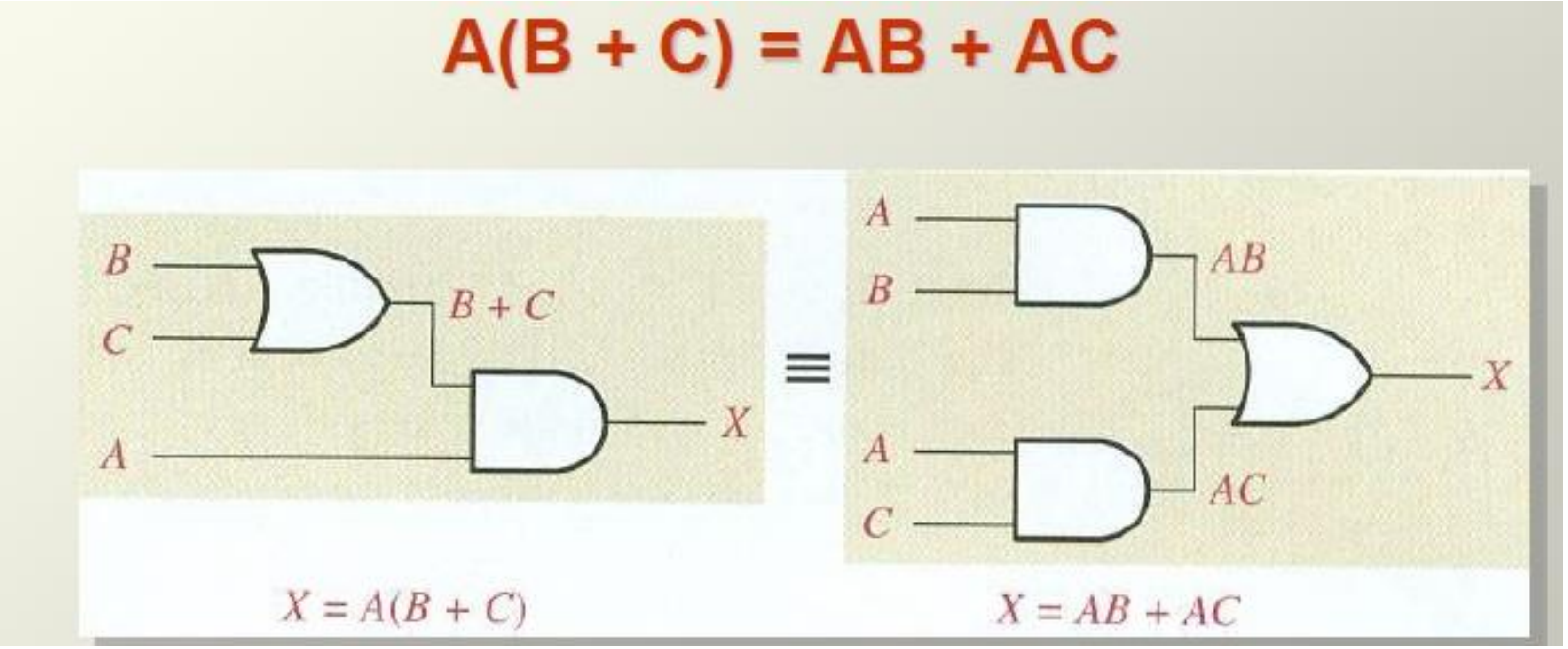
Associative Law (Addition)



Associative Law (Multiplication)



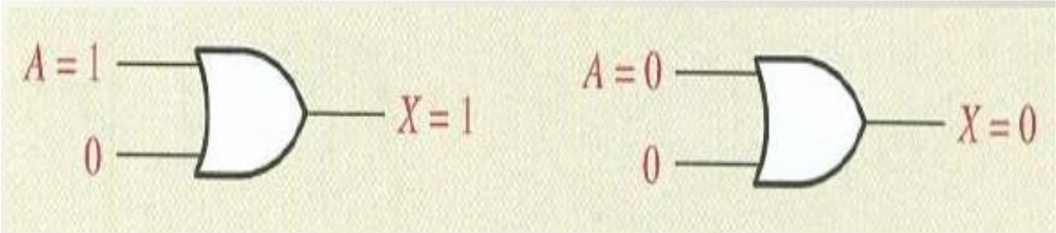
Distributive Law



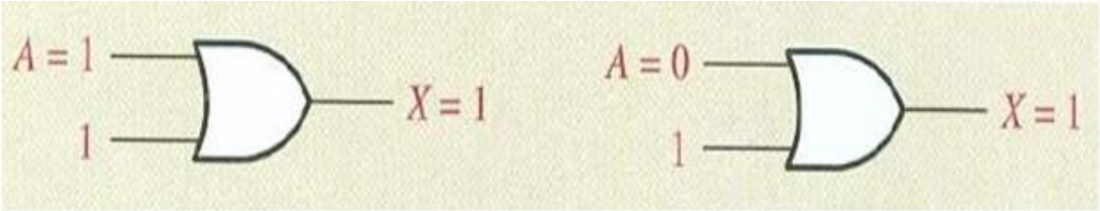
Rules of Boolean Algebra

Rules of Boolean Algebra

Rule 1 Identity (ADDITION): X=A+0=A



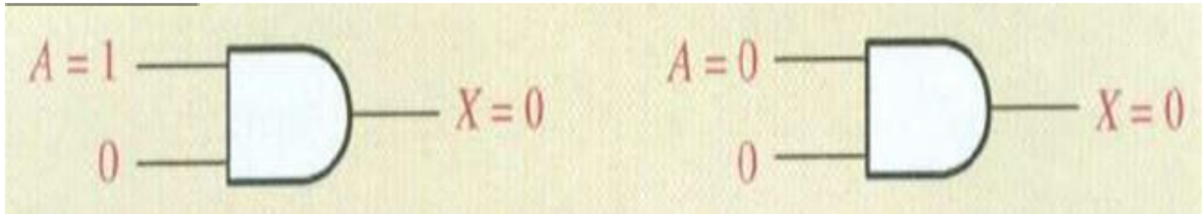
Rule 2 NULL (ADDITION): X=A+1=1



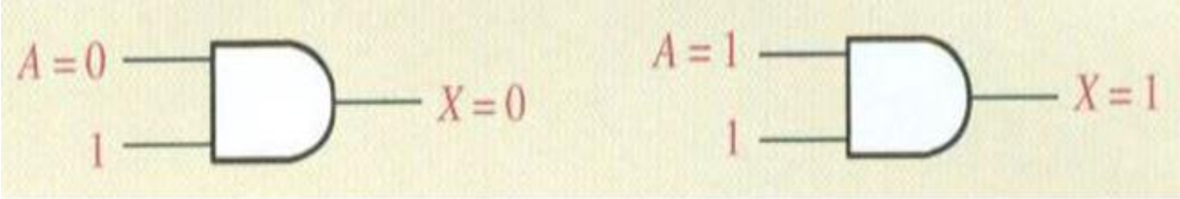
A	B	X
0	0	0
0	1	1
1	0	1
1	1	1

Rules of Boolean Algebra

Rule 3 NULL (MULTIPLICATION): X=A∗0=0



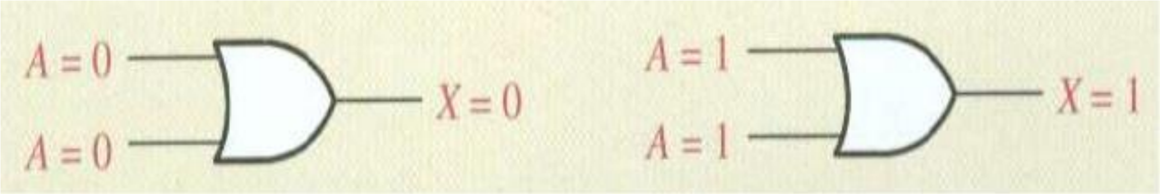
Rule 4 Identity (MULTIPLICATION): X=A∗1=A



A	B	X
0	0	0
0	1	0
1	0	0
1	1	1

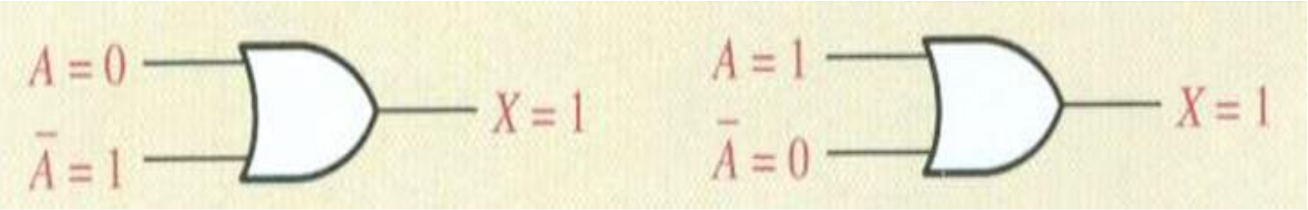
Rules of Boolean Algebra

Rule 5 Idempotent (ADDITION): X=A+A=A



A	B	X
0	0	0
1	1	1

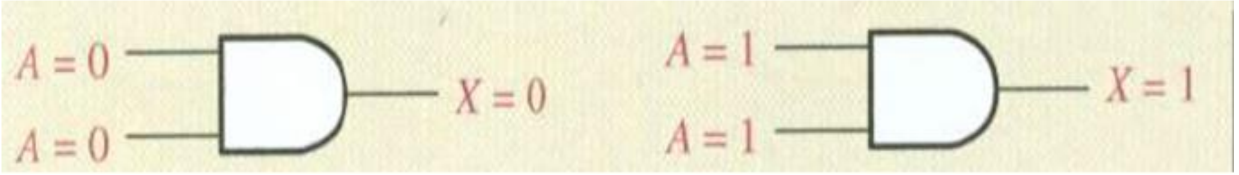
Rule 6 Complementary (ADDITION): $X=A+\overline{A}=1$



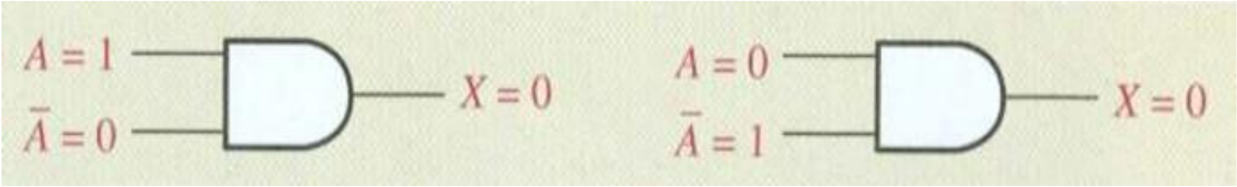
A	B	X
0	0	0
0	1	1
1	0	1
1	1	1

Rules of Boolean Algebra

Rule 7 Idempotent (MULTIPLICATION): X= A$*$A=A

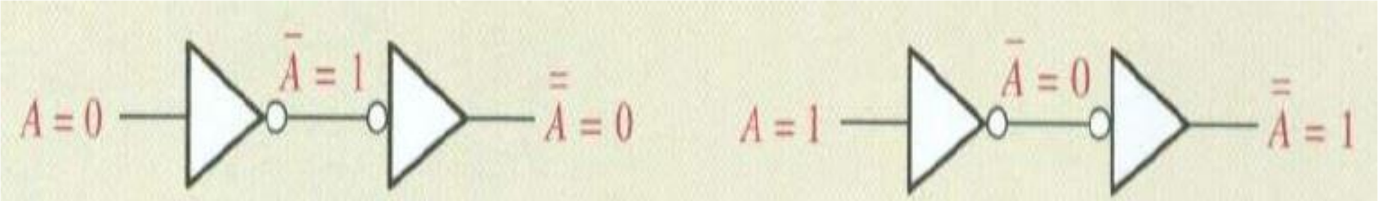


Rule 8 Complementary (MULTIPLICATION): X= A$*\bar{A}$=0



Rules of Boolean Algebra

Rule 9 Involution: A=$\bar{\bar{A}}$

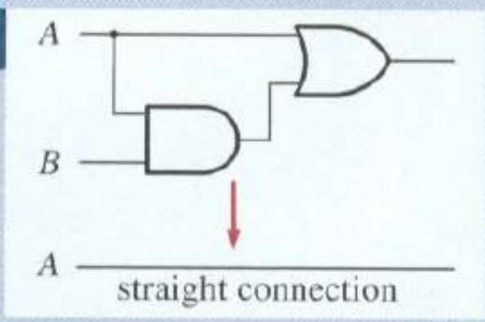


Rules of Boolean Algebra

Rule 10 OR Absorption: A + (A • B) = A

A	B	AB	A + AB
0	0	0	0
0	1	0	0
1	0	0	1
1	1	1	1

equal



The proof:
Left side = A + AB

= A (1+B)

=A•1

=A

Rules of Boolean Algebra

Rule 11 AND Absorption: A • (A + B) = A

The proof:

Left side = A(A+B)

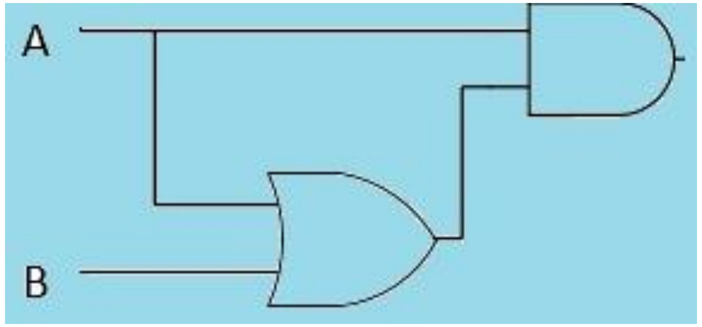
= A•A+AB

= A + AB

= A(1 + B)

=A•1

=A



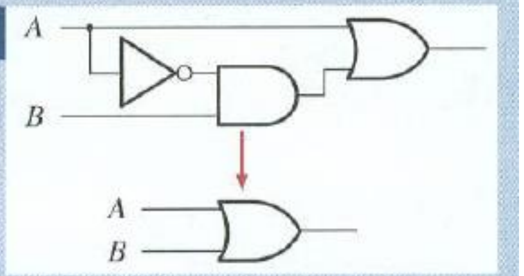
A	B	A+B	A.(A+B)
0	0	0	0
0	1	1	0
1	0	1	1
1	1	1	1

Rules of Boolean Algebra

Rule 12 Redundancy: $A + \bar{A}\,B = (A + B)$

A	B	$\bar{A}B$	$A + \bar{A}B$	$A + B$
0	0	0	0	0
0	1	1	1	1
1	0	0	1	1
1	1	0	1	1

equal



The proof:

Left side = $A+\bar{A}B$

$= (A + \bar{A})\,(A+B)$

$= 1 * (A+B)$

$=A+B$

= Right side

Rules of Boolean Algebra

Rule 13: (A + B)(A + C) = A + BC

The proof:

Left side = (A+B)(A+C)

= AA + AC + AB + BC

= A + AC + AB + BC

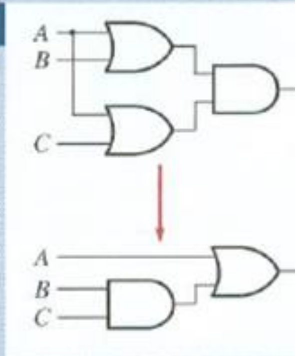
= A(1 + C + B) + BC

= A • 1 + BC

= A + BC = Right side

A	B	C	A + B	A + C	(A + B)(A + C)	BC	A + BC
0	0	0	0	0	0	0	0
0	0	1	0	1	0	0	0
0	1	0	1	0	0	0	0
0	1	1	1	1	1	1	1
1	0	0	1	1	1	0	1
1	0	1	1	1	1	0	1
1	1	0	1	1	1	0	1
1	1	1	1	1	1	1	1

equal

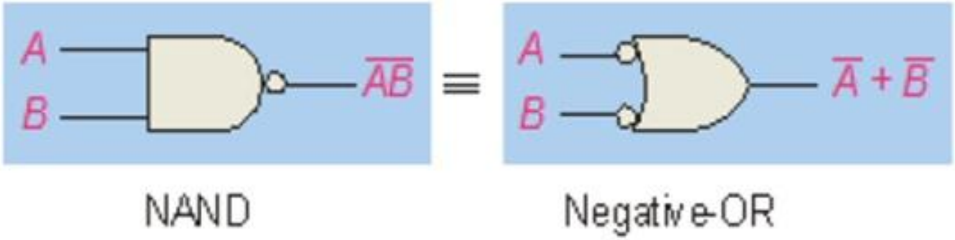


DeMorgan's Theorem (First Theorem)

The complement of a product of variables is equal to the sum of complemented variables.

$\overline{AB} = \bar{A} + \bar{B}$

Applying DeMorgan's First Theorem to gates:

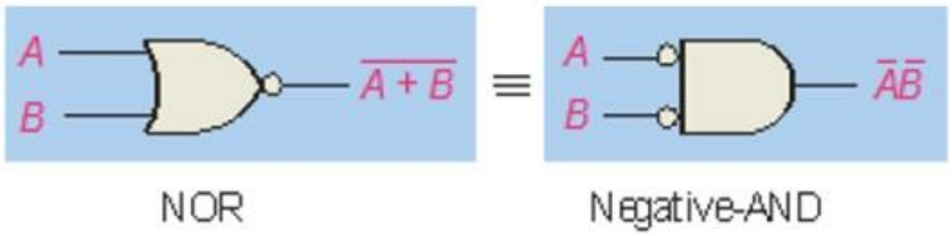


A	B	~ A	~ B	~ AB	~ A + ~B
0	0	1	1	1	1
0	1	1	0	1	1
1	0	0	1	1	1
1	1	0	0	0	0

DeMorgan’s Theorem (Second Theorem)

The complement of a sum of variables is equal to the product of complemented variables. $\overline{A+B}=\bar{A}\bullet\bar{B}$

Applying DeMorgan’s Second Theorem to gates:



A	B	~ A	~ B	~ (A+B)	~ A . ~B
0	0	1	1	1	1
0	1	1	0	0	0
1	0	0	1	0	0
1	1	0	0	0	0

Example

Example: Apply DeMorgan's theorem to simplify the following expression.

X=$\overline{\bar{c}+D}$

To apply DeMorgan's theorem to the above expression, we can break the over-bar that covering both terms and change the sign between them as follows:

X=$\bar{\bar{C}}$• $\bar{D}$

X=C• $\bar{D}$

Example

Example: Simplify the following expression

Y =(A+B)(A+$\bar{B}$)

= AA + A$\bar{B}$ + AB + B$\bar{B}$

= A + A($\bar{B}$ + B)

=A+A

=A

OR:

Y =AA+A$\bar{B}$+AB+B$\bar{B}$

= AA + A$\bar{B}$ + AB

= A + A$\bar{B}$ + AB

= A(1 + $\bar{B}$ + B)

=A•1

=A

Example

Example:

Y =ABC+A$\bar{B}$C+AB$\bar{C}$

= AB(C + $\bar{C}$) + A$\bar{B}$C

= AB + A$\bar{B}$C

= A(B + $\bar{B}$C)

= A(B + C)

Example:

Y = ABC + A $\bar{B}$ $(\overline{\bar{A}\ \bar{C}})$

= ABC + A$\bar{B}$ $(\bar{\bar{A}} + \bar{\bar{C}})$

= ABC + A$\bar{B}$ (A + C)

= ABC + A $\bar{B}$ + A$\bar{B}$C

= AC(B + $\bar{B}$) + A$\bar{B}$

= AC + A $\bar{B}$

= A (C +$\bar{B}$)

Example

Determine if the following equation is valid (Derived from the truth table)

$\bar{X}1\ \bar{X}3+X2\ X3+X1\ \bar{X}2 = \bar{X}1\ X2+X1\ X3+\bar{X}2\ \bar{X}3$

Left-Hand Side (LHS)

Row number	x_1	x_2	x_3	$\bar{x}_1 \bar{x}_3$	$x_2 x_3$	$x_1 \bar{x}_2$	f
0	0	0	0	1	0	0	1
1	0	0	1	0	0	0	0
2	0	1	0	1	0	0	1
3	0	1	1	0	1	0	1
4	1	0	0	0	0	1	1
5	1	0	1	0	0	1	1
6	1	1	0	0	0	0	0
7	1	1	1	0	1	0	1

Right-Hand Side (RHS)

Row number	x_1	x_2	x_3	$\bar{x}_1 x_2$	$x_1 x_3$	$\bar{x}_2 \bar{x}_3$	f
0	0	0	0	0	0	1	1
1	0	0	1	0	0	0	0
2	0	1	0	1	0	0	1
3	0	1	1	1	0	0	1
4	1	0	0	0	0	1	1
5	1	0	1	0	1	0	1
6	1	1	0	0	0	0	0
7	1	1	1	0	1	0	1

Example:

Simplify the following expression, then draw the logic circuit after simplification.

Y =$\overline{A + B}$+$\overline{ABC}$+$\overline{A\bar{B}}$

=$\bar{A}\ \bar{B}$ + $\bar{A}$ + $\bar{B}$ + $\bar{C}$ + $\bar{A}$ +B

= $\bar{A}$($\bar{B}$ + 1 + 1) + (B + $\bar{B}$) + $\bar{C}$

= $\bar{A}$ + 1 + $\bar{C}$

=1

logic circuit= Nothing

Example:

For the given truth table simplify and draw the circuit before and after simplification.

$Y = \bar{A}B\bar{C} + \bar{A}BC + AB\bar{C} + ABC$

$= \bar{A}B(\bar{C} + C) + AB(\bar{C} + C)$

$= \bar{A}B + AB$

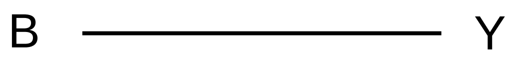
$= B(\bar{A} + A)$

$= B$

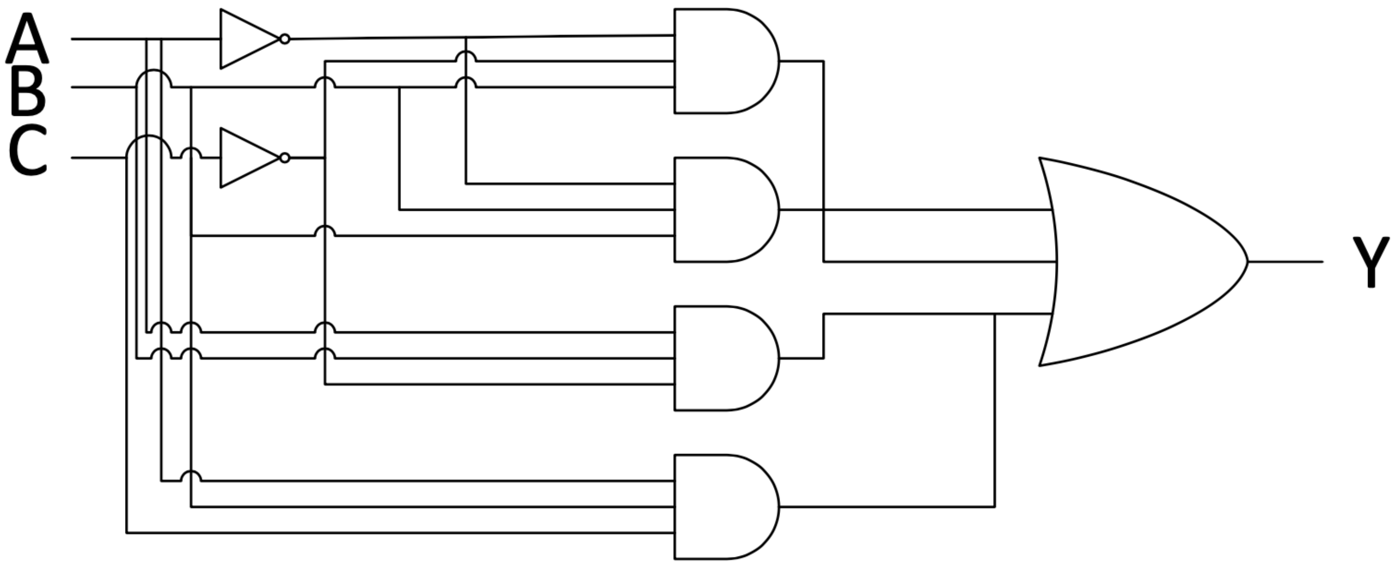
A	B	C	Y
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	1
1	1	1	1

Drawing of the circuit before and after simplification

The circuit after simplification:



The circuit before simplification:



Sum of Products

The *Sum of Product* (SOP) expression comes from the fact that two or more products (AND) are summed (OR) together. That is the outputs from two or more AND gates are connected to the input of an OR gate so that they are effectively OR'ed together to create the final AND-OR logical output.

$$Q = (A.B) + (\bar{B}.C) + (A.1)$$

However, Boolean functions can also be expressed in nonstandard sum of products forms like that shown below but they can be converted to a standard SOP form by expanding the expression. So:

$$Q = A.\bar{B}(\bar{C} + C) + ABC$$

Becomes in sum-of-product terms:

$$Q = A.\bar{B}.\bar{C} + A.\bar{B}.C + ABC$$

Sum-of-Product Example

The following Boolean Algebra expression is given as:

Q = $\bar{A}(\bar{B}$C + BC + B$\bar{C}$) + ABC

1. Convert this logical equation into an equivalent SOP term.

2. Use a truth table to show all the possible combinations of input conditions that will produces an output.

3. Draw a logic gate diagram for the expression.

Sum-of-Product Example

1. Convert to SOP term

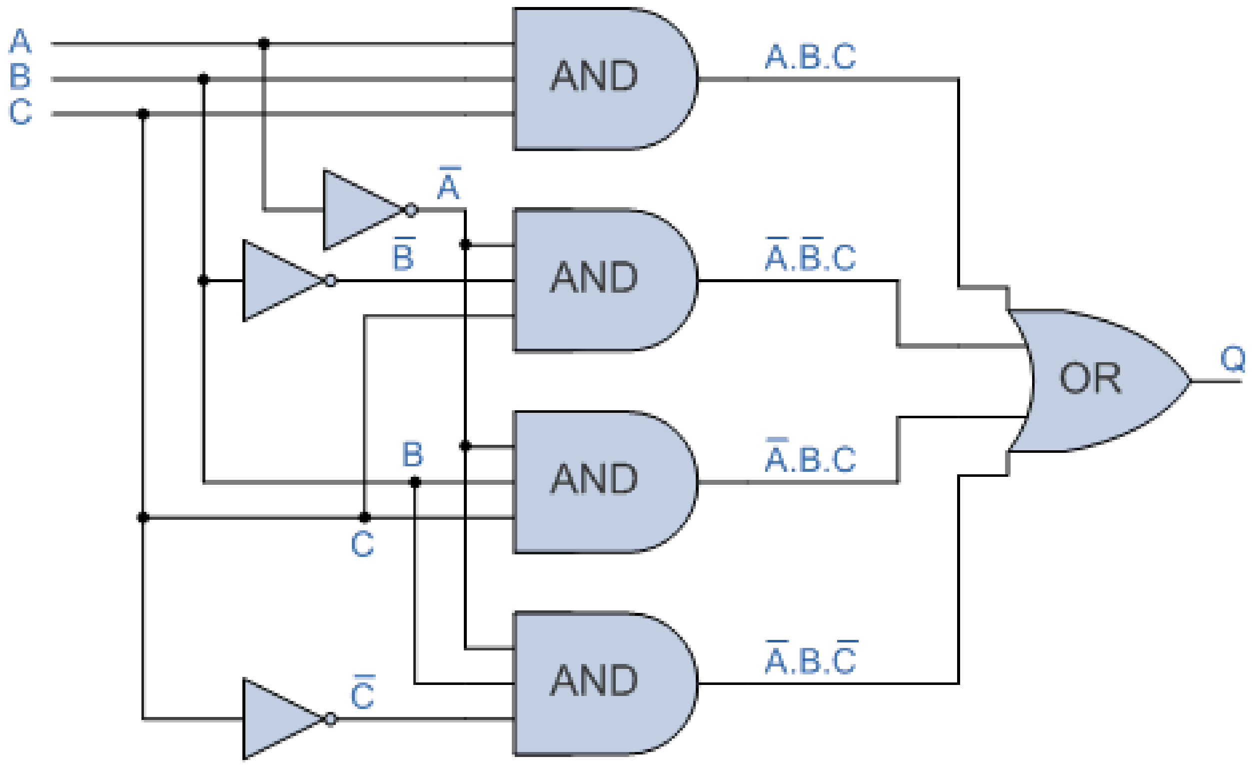
Q = A.B.C + $\bar{A}.\bar{B}$.C + $\bar{A}$.B.C + $\bar{A}$.B.$\bar{C}$

2. Truth Table: **Sum of Product Truth Table Form**

inputs			output	Product
A	B	C	Q	
0	0	0	0	
0	0	1	1	$\bar{A}.\bar{B}.C$
0	1	0	1	$\bar{A}.B.\bar{C}$
0	1	1	1	$\bar{A}.B.C$
1	0	0	0	
1	0	1	0	
1	1	0	0	
1	1	1	1	$A.B.C$

Sum-of-Product Example

3. Logic Gate SOP Diagram



Product of Sum (POS)

The *Product of Sum* (POS) expression comes from the fact that two or more sums (OR's) are added (AND'ed) together. That is the outputs from two or more OR gates are connected to the input of an AND gate so that they are effectively AND'ed together to create the final (OR AND) output.

Product of Sum Expressions: $Q = (A + B).(B + C).(A + 1)$

$Q = A + (BC)$ → using the distributive law : $Q = (A + B)(A + C)$

$Q = (A + B) + (A.C)$ → $Q = (A + B + A)(A + B + C)$

Product of Sum example

The following Boolean Algebra expression is given as:

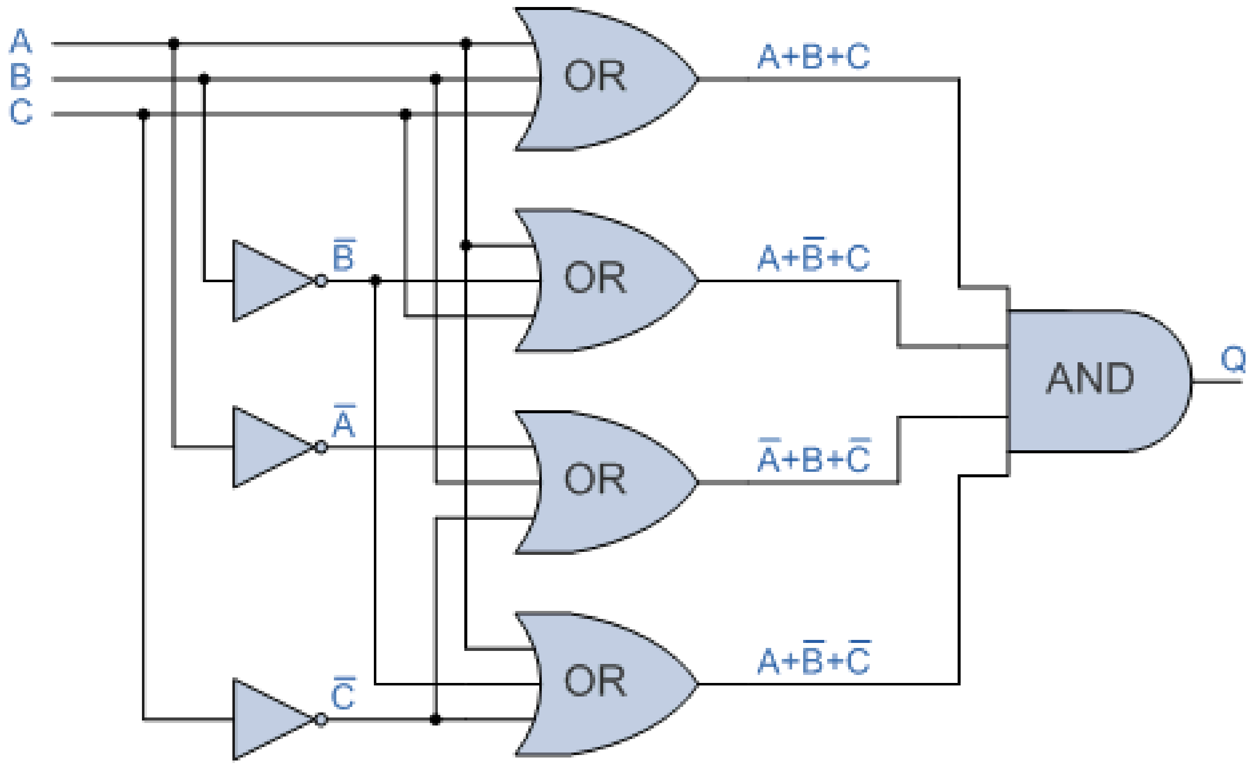
$$Q = (A + B + C)(A + \bar{B} + C)(A + \bar{B} + \bar{C})(\bar{A} + B + \bar{C})$$

1. Use a truth table to show all the possible combinations of input conditions that will produces a “0” output.

2. Draw a logic gate diagram for the POS expression.

Example

inputs			output	Product
A	B	C	Q	
0	0	0	0	A+B+C
0	0	1	1	
0	1	0	0	$A+\bar{B}+C$
0	1	1	0	$A+\bar{B}+\bar{C}$
1	0	0	1	
1	0	1	0	$\bar{A}+B+\bar{C}$
1	1	0	1	
1	1	1	1	



Homework

1- Design a logic circuit with three inputs variables (A, B, C) that will produce a HIGH ("1") output if the input is Odd number?

2- Design a logic circuit that has three inputs, *A*, *B*, and *C*, and whose output will be HIGH only when two or more inputs are HIGH.

References

https://whatis.techtarget.com/definition/logic-gate-AND-OR-XOR-NOT-NAND-NOR-and-XNOR

http://www.ee.surrey.ac.uk/Projects/CAL/digital-logic/gatesfunc/#andgate

محاضرات المنطق / د. فارس ٢٠٢١

https://www.youtube.com/watch?v=JbIBDCTWAnQ