Logical Design

Lecture 11

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In this lesson

- Karnaugh Map definition.
- Advantages and Disadvantages of Karnaugh Map.
- Grouping rules of K-map variables.
- Examples

Karnaugh Map

- A Karnaugh map (K-map) is a method for minimising Boolean expressions without having to apply Boolean algebra theorems and equation manipulations by translating a truth table to its equivalent logic circuit in a simple orderly process, as presented by Maurice Karnaugh in 1953.

Advantages and Disadvantages of Karnaugh Map

Advantages of K-map are:

1. K-map simplification does not demand for the knowledge of Boolean algebraic theorems.
2. It requires a smaller number of steps when compared to algebraic minimization technique.
3. It is easy to convert a truth table to k-map and k-map to Sum of Products form equation.
4. The K-map method is faster and more efficient than other simplification techniques of Boolean algebra.

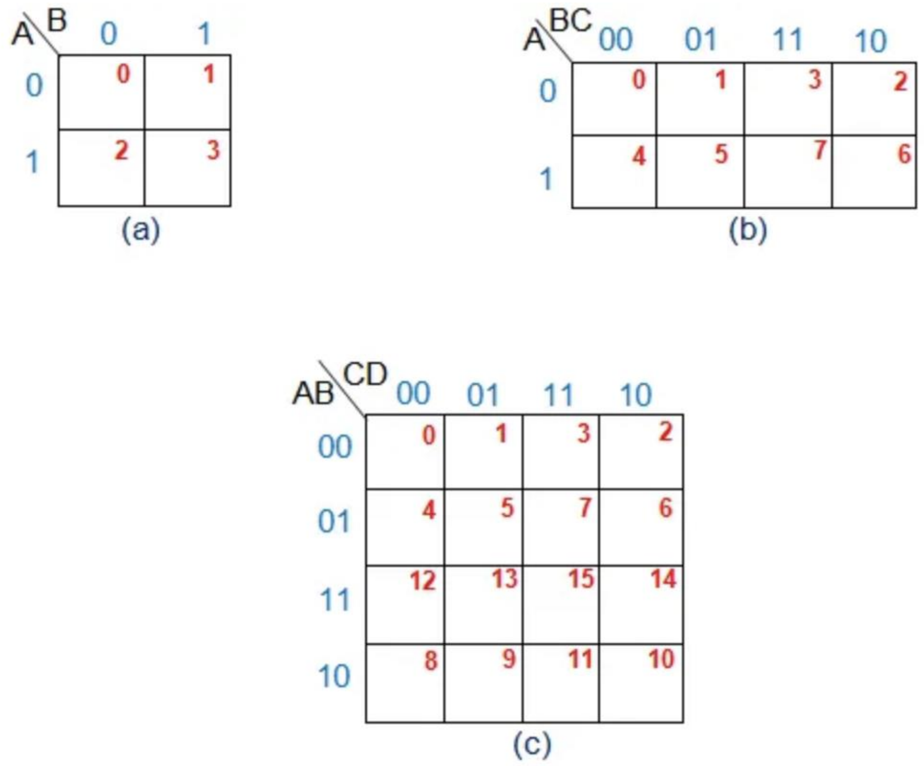
Disadvantages of K-map are:

1. Complexity of **K-map** simplification process increases with the increase in the number of variables
2. The minimum expression obtained might not be unique

Karnaugh Map

In general, if there are n inputs, then the corresponding K-map has to be of 2^n cells.
For example:

- if the number of input variables is **2**, then we have to consider a K-map with 4 (=2^2) cells
- while if there are **3** input variables, then we require an 8 (=2^3) cell K-map,
- similarly for **4** inputs one gets 16 (=2^4) cell K-map and so on.



Karnaugh Maps for (a) Two Variables (b) Three Variables (c) Four Variables

Exapmle – two variable map

For the following truth table:

P	Q	output
0	0	A
0	1	B
1	0	C
1	1	D

The K-map for the truth table:

P \ Q	0	1
0	A	B
1	C	D

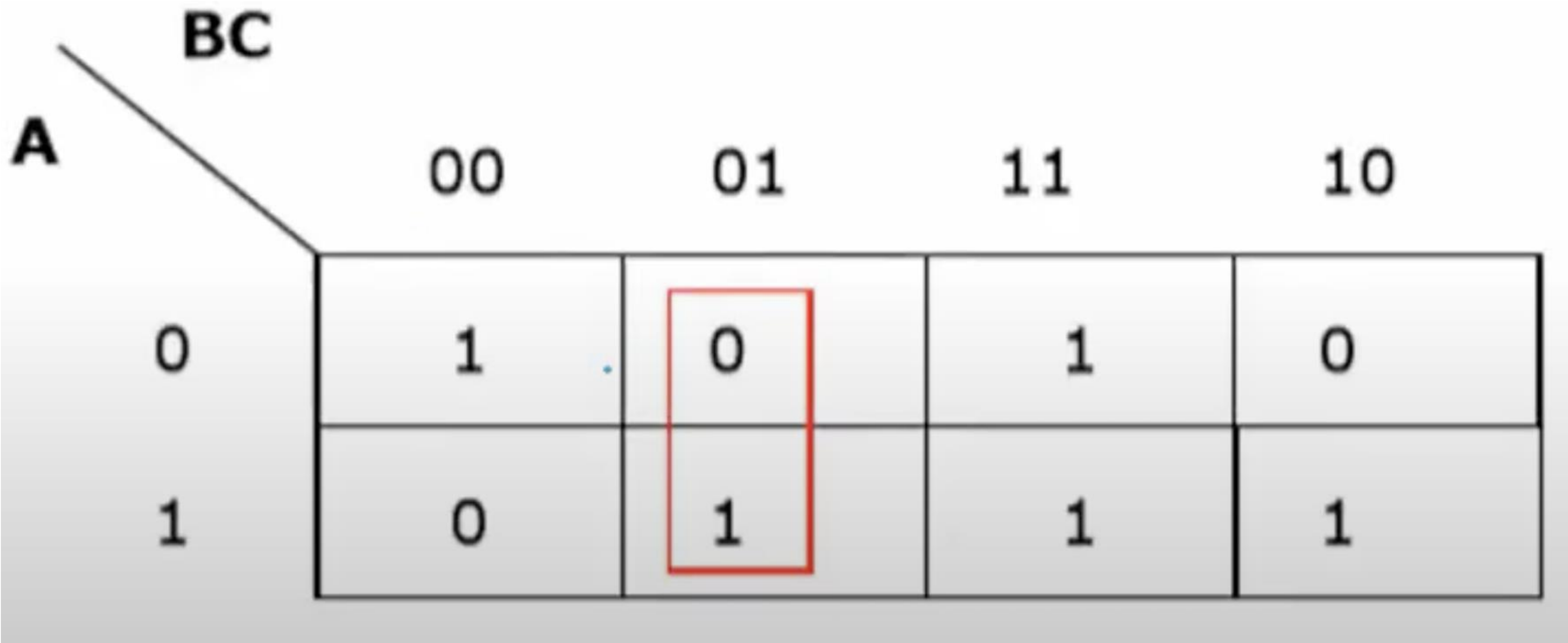
Exapmle – three variable map

x_1	x_2	x_3	
0	0	0	m_0
0	0	1	m_1
0	1	0	m_2
0	1	1	m_3
1	0	0	m_4
1	0	1	m_5
1	1	0	m_6
1	1	1	m_7

x_3 \ $x_1 x_2$	00	01	11	10
0	m_0	m_2	m_6	m_4
1	m_1	m_3	m_7	m_5

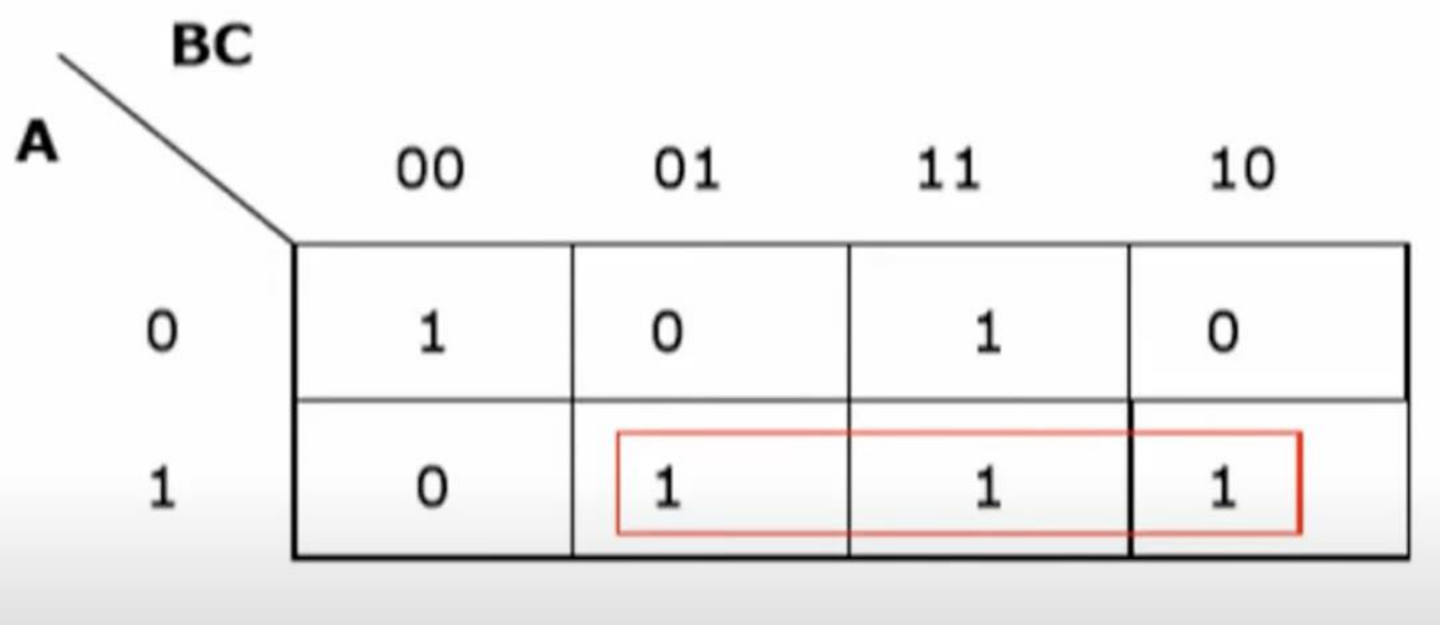
K-map Variables Grouping Rules

Rule 1: Groupe should not contains any Zeros



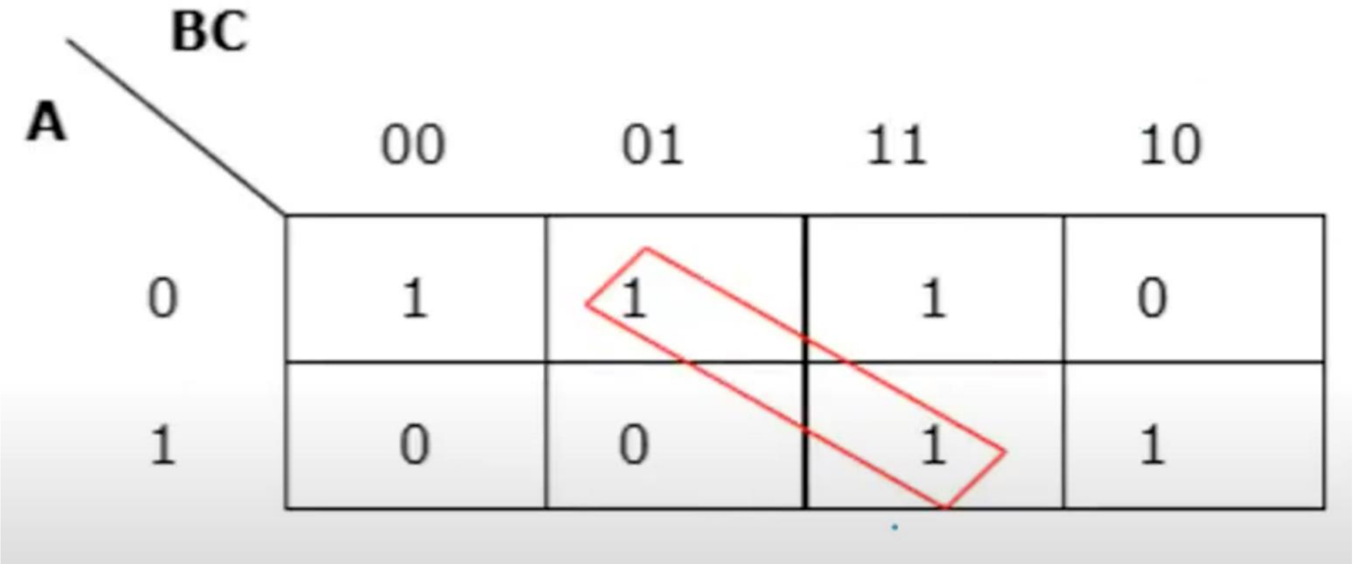
K-map Variables Grouping Rules

Rule 2: Groups must contain 2^n cells (n = 0,1,2,3, …)



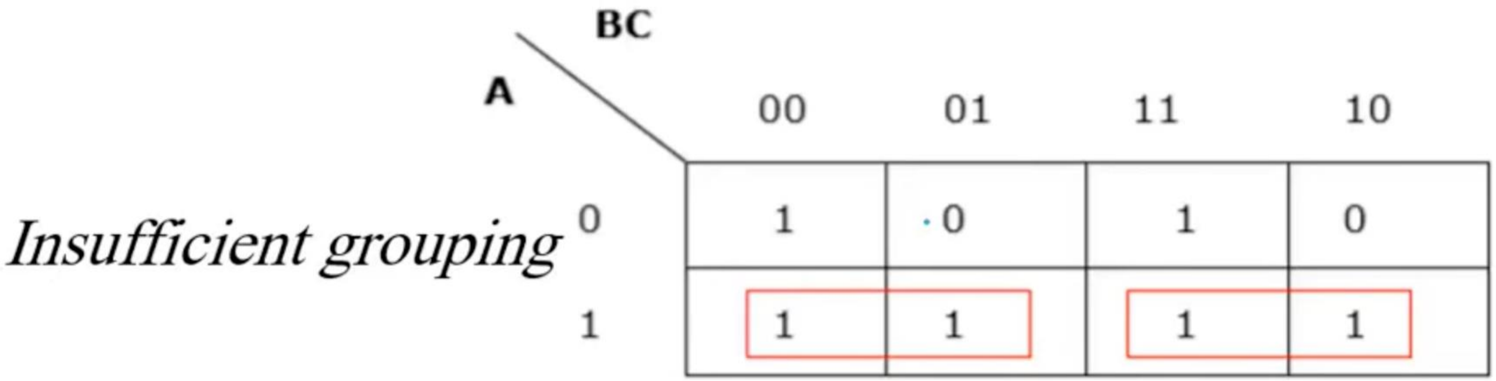
K-map Variables Grouping Rules

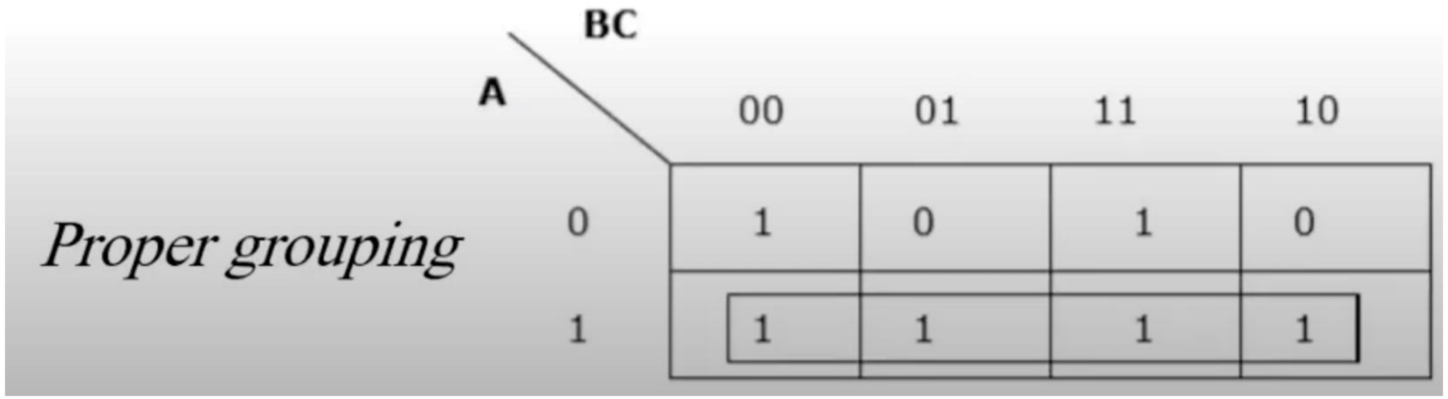
Rule 3: Grouping must be horizontal or vertical but not diagonal.



K-map Variables Grouping Rules

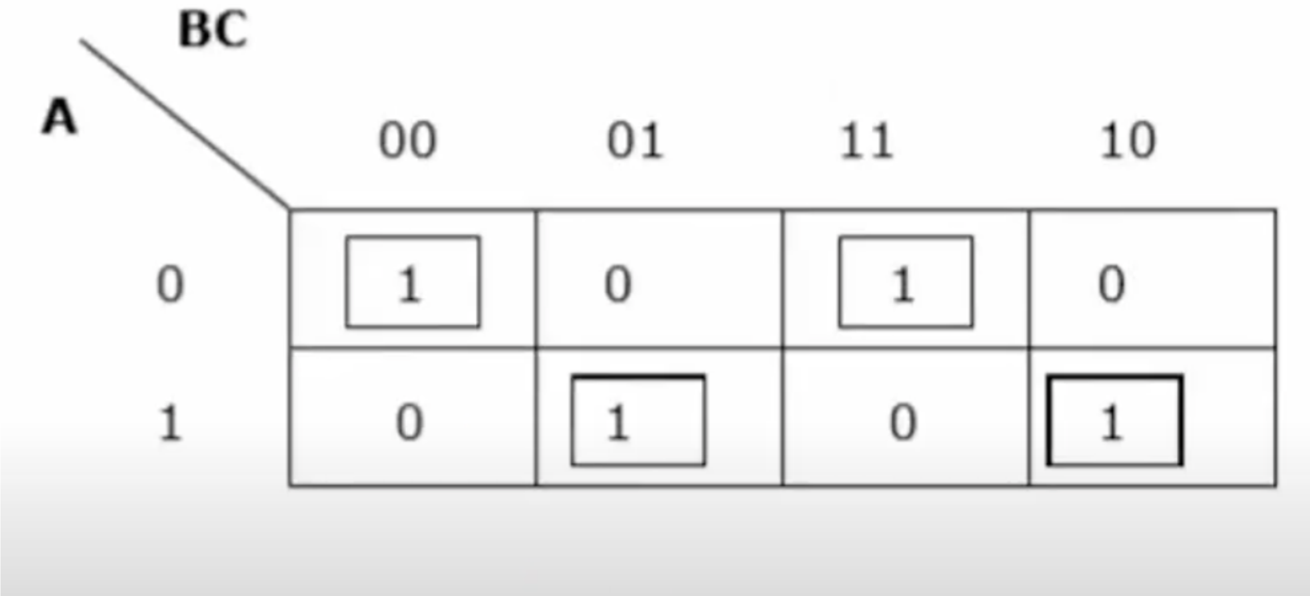
Rule 4: Groups must cover as large as possible





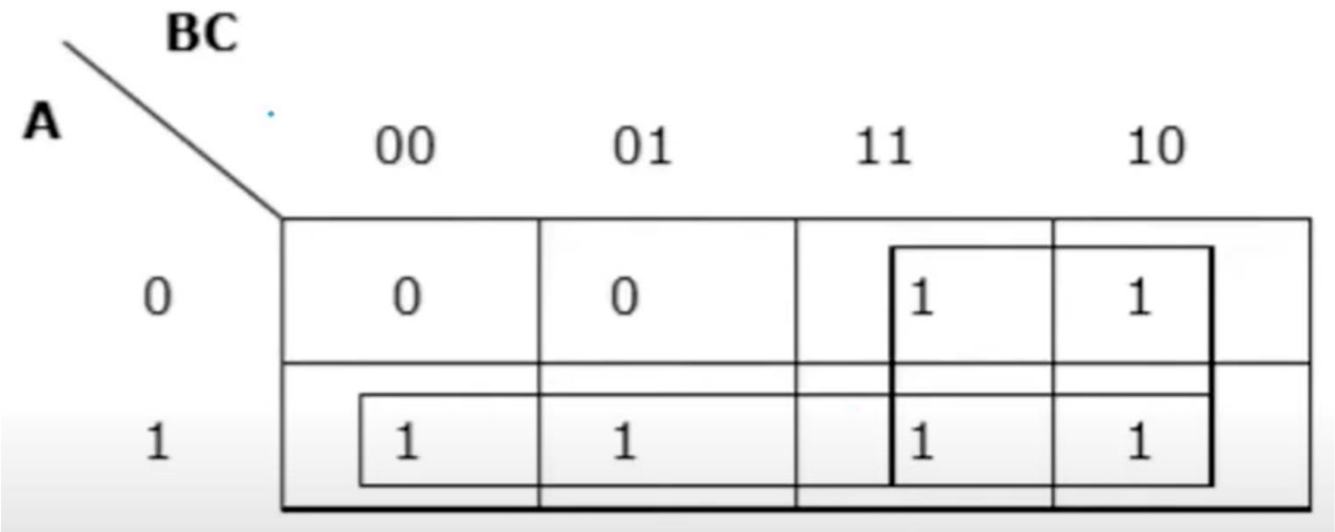
K-map Variables Grouping Rules

Rule 5: if 1 of any cell cannot be grouped with any other cell, then it will act as a group itself



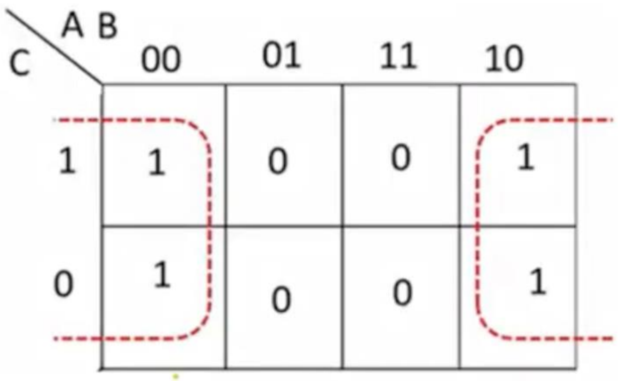
K-map Variables Grouping Rules

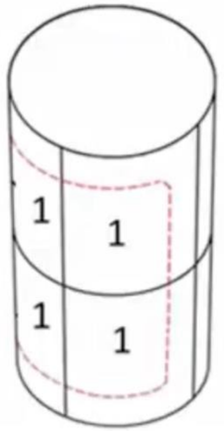
Rule 6: Groups may overlap but there should be as few groups as possible.



K-map Variables Grouping Rules

Rule 7: The leftmost cell/ cells can be grouped with the rightmost cell/cells and the topmost cell/ cells can be grouped with the bottommost cell/cells.





Example

Simplify the following boolean expression using k-map method:

F(A,B,C) = $\bar{A}BC + \bar{A}\,B\,\bar{C} + A\,\bar{B}\,\bar{C} + A\,\bar{B}C$

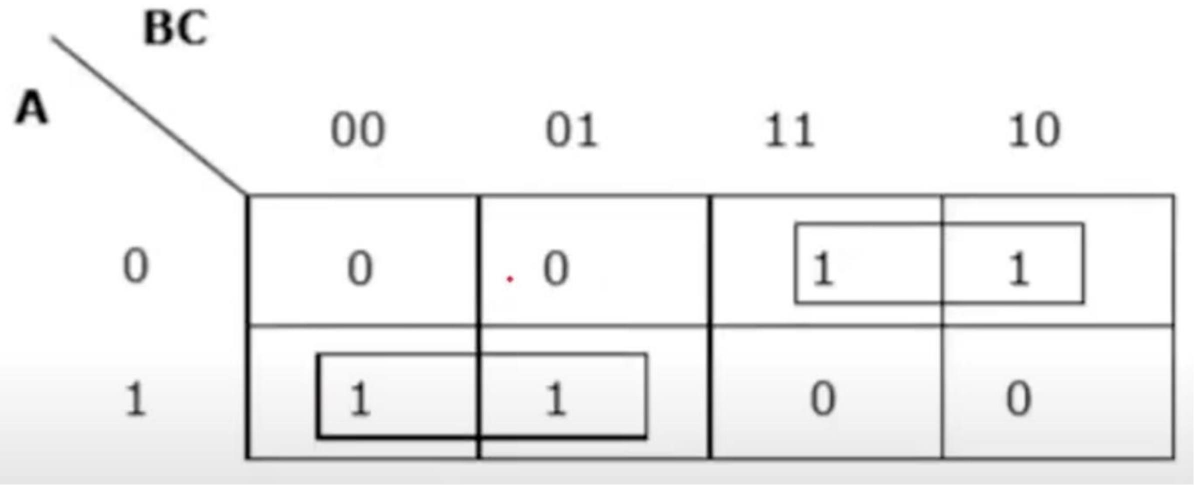
Solution: Creat the truth table first then k-map

Inputs			Output
A	B	C	F
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	0
1	1	1	0

A \ BC	00	01	11	10
0	0	0	1	1
1	1	1	0	0

Example – continoued

Now the 1s will be gouped accourding the above rules



F(A,B,C) = $\bar{A}B + A\bar{B}$

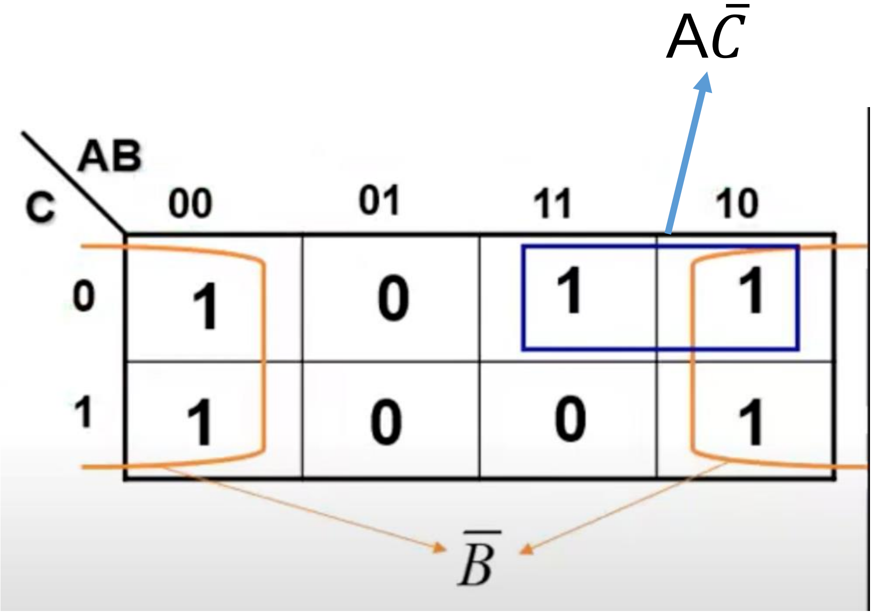
Exapmle

Using k-map to derive the minimal SOP for the output Y(A,B,C) of the given truthtable below:

Inputs			Output
A	B	C	Y
0	0	0	1
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	0

C \ AB	00	01	11	10
0	1	0	1	1
1	1	0	0	1

Example – continoued



$Y = \bar{B} + A\,\bar{C}$

4 variables K-maps

Number of Cells = 2^4=16

CD \ AB	00	01	11	10
00	0	4	12	8
01	1	5	13	9
11	3	7	15	11
10	2	6	14	10

Exapmle

A	B	C	D	*F*
0	0	0	0	0
0	0	0	1	0
0	0	1	0	1
0	0	1	1	1
0	1	0	0	1
0	1	0	1	1
0	1	1	0	1
0	1	1	1	1
1	0	0	0	1
1	0	0	1	1
1	0	1	0	0
1	0	1	1	1
1	1	0	0	1
1	1	0	1	0
1	1	1	0	0
1	1	1	1	1

Using k-map to derive the minimal SOP for the output F(A,B,C,D) of the given truthtable:

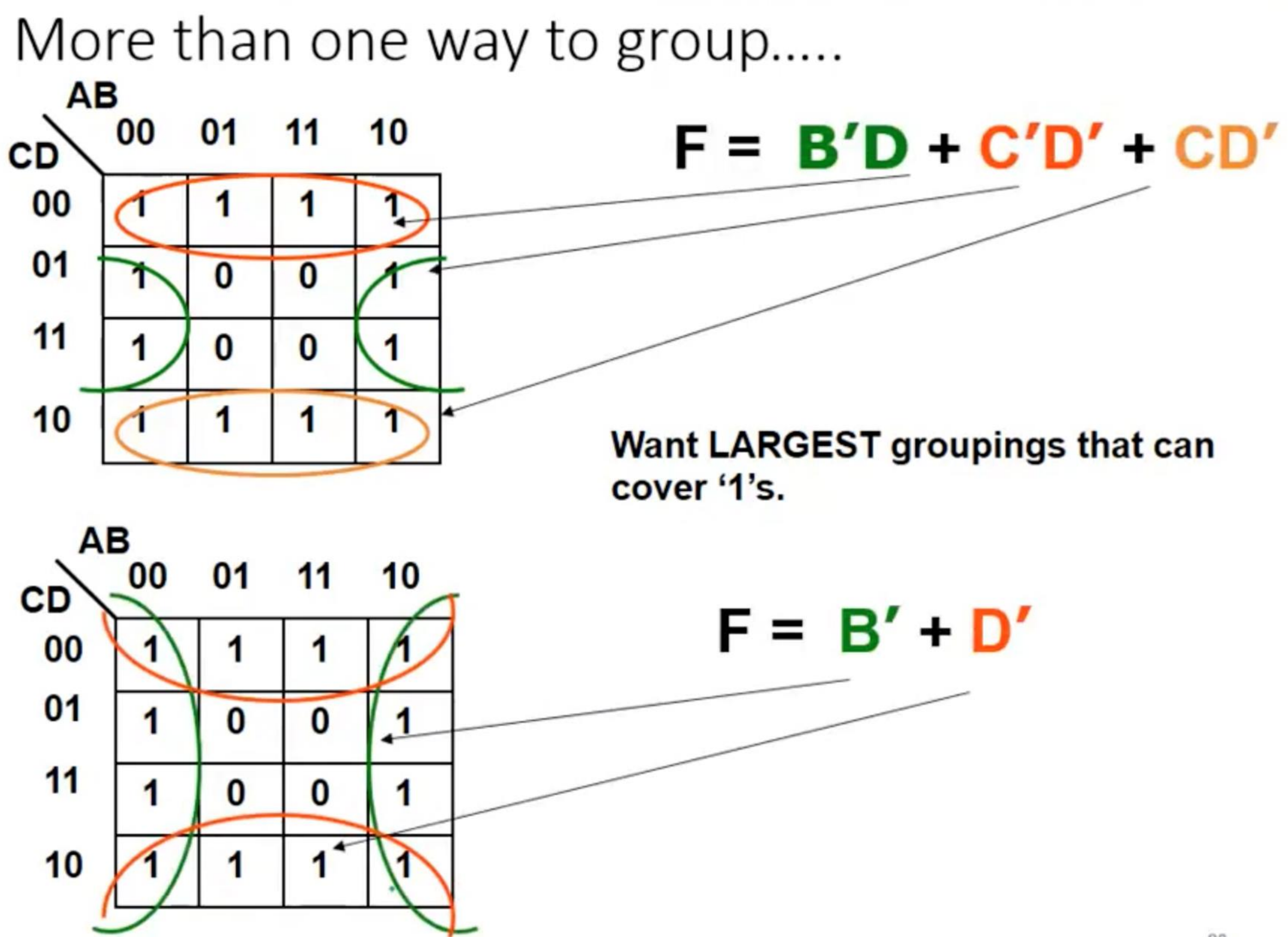
CD \ AB	00	01	11	10
00	0	1	1	1
01	0	1	0	1
11	1	1	1	1
10	1	1	0	0

Example – continoued

CD \ AB	00	01	11	10
00	0	1	1	1
01	0	1	0	1
11	1	1	1	1
10	1	1	0	0

$$F = \bar{A}C + CD + \bar{A}B + A\bar{B}\bar{C} + A\bar{C}\bar{D}$$

Example of different types of grouping



Summary of Grouping Rules

- A group can only be horizontal or vertical, not diagnonal
- A group must contain 2n of once (1,2,3,8, ….)
- Each group shouls be as large as possible
- Groups may overlap
- Groups may warp around a table
- Goups should be as few as possible

Home work

Example: Using K-map, derive minimal SOP for the output Y(A,B,C) whose truth table is given below:

A	B	C	Y
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1

Home work

Example: for the following K-map, derive minimal SOP.