Logical Design Lectures

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Lecture 1 – Number Systems

- In early days when there were no tools of counting, people use to count with the help of fingers, stones, sticks, etc.
- Number systems are useful in digital computers and the knowledge of these systems is necessary to perform reliable, economic and easily understandable arithmetic operation
- Generally there is an order set of symbols that known as "Digit"
- Two types of number systems are:
  - Non- positional number systems: Use symbols such as I for 1, II for 2, III for 3, IV for 4, V for 5, etc. Each symbol represents the same value regardless of its position in the number. The symbols are simply added to find out the value of a particular number Positional number systems.
  - Positional number systems: Use only a few symbols called digits, these symbols represent different values depending on the position they occupy in the number. The value of each digit is determined by the digit itself, the position of the digit in the number, the base of the number system (base = total number of digits in the number system). The maximum value of a single digit is always equal to one less than the value of the base.
- A number system is a collection of various symbols which are called "Digit". A real number has two parts, Integer part and Fractional part, separated by a a radix point "." which known as the decimal point when the decimal

number is used.

★ 1- Decimal Number System : A positional number system

- Has 10 symbols or digits (0, 1, 2, 3, 4, 5, 6, 7, 8, 9). Hence, its base = 10.
- The maximum value of a single digit is 9 (one less than the value of the base).
- Each position of a digit represents a specific power of the base (10).
- We deal with this number system in our life everyday.
- The absolute value of each digit is fixed, but its positional value is determined by its relative position or place in a given number, where this positional value is known as Weight. See the following form:

$$\underbrace{\ldots + n*10^3 + n*10^2 + n*10^1 + n*10^0}_{\textbf{INTEGER}} \bullet \underbrace{n*10^{-1} + n*10^{-2} + n*10^{-3} + \ldots\ldots}_{\textbf{FRACTION}}$$

Example:
$456.7 = 4*10^2 + 5*10^1 + 6*10^0 \bullet 7*10^{-1}$

$= (456.7)_{10}$

★ 2- Binary Number System : A positional number system

- Has only 2 symbols or digits (0 and 1). Hence its base = 2
- The maximum value of a single digit is 1 (one less than the value of the base).
- Each position of a digit represents a specific power of the base (2).
- This number system is used in the computers.
- The absolute value of each digit is fixed, but its positional value is determined by its relative position or place in a given number, where this positional value is known as Weight. See the following form:

$$\underbrace{\ldots + n*2^3 + n*2^2 + n*2^1 + n*2^0}_{\textbf{INTEGER}} \bullet \underbrace{n*2^{-1} + n*2^{-2} + n*2^{-3} + \ldots\ldots}_{\textbf{FRACTION}}$$

where n equal "0" or "1"

Example:

$10101_2 = 1*2^4 + 0*2^3 + 1*2^2 + 0*2^1 + 1*2^0$

$= 16 + 0 + 4 + 0 + 1$

$= (21)_{10}$

★ 3- Octal Number System : A positional number system

- Has total 8 symbols or digits (0, 1, 2, 3, 4, 5, 6, 7). Hence, its base = 8.
- The maximum value of a single digit is 7 (one less than the value of the base).
- This number system is used in the computers.
- Each position of a digit represents a specific power of the base (8).
- The absolute value of each digit is fixed, but its positional value is determined by its relative position or place in a given number, where this positional value is known as Weight. See the following form:

$$\underbrace{\ldots + n*8^3 + n*8^2 + n*8^1 + n*8^0}_{\textbf{INTEGER}} \bullet \underbrace{n*8^{-1} + n*8^{-2} + n*8^{-3} + \ldots\ldots}_{\textbf{FRACTION}}$$

Example:

$2057_8 = 2*8^3 + 0*8^2 + 5*8^1 + 7*8^0$

$= 1024 + 0 + 4 + 7$

$= (1071)_{10}$

★ 4- Hexadecimal Number System : A positional number system

- Has total 16 symbols or digits (0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F). Hence its base = 16.
- The symbols A, B, C, D, E and F represent the decimal values 10, 11, 12, 13, 14 and 15 respectively.
- The maximum value of a single digit is 15 (one less than the value of the base).
- Each position of a digit represents a specific power of the base (16).
- Since there are only 16 digits, 4 bits ($2^4 = 16$) are sufficient to represent any hexadecimal number in the binary system.
- The absolute value of each digit is fixed, but its positional value is determined by its relative position or place in a given number, where this positional value is known as Weight. See the following form:

$$\underbrace{\ldots + n * 16^3 + n * 16^2 + n * 16^1 + n * 16^0}_{\textbf{INTEGER}} \bullet \underbrace{n * 16^{-1} + n * 16^{-2} + n * 16^{-3} + \ldots\ldots}_{\textbf{FRACTION}}$$

Example:

$1AF_{16} = 1 * 16^2 + A * 16^1 + F * 16^0$

$= 1 * 256 + 10 * 16 + 15 * 1$

$= 256 + 160 + 15$

$= (431)_{10}$

★ 5- Converting a Number of Binary or Octal or Hexadecimal Base to a Decimal Number. We need to follow the following steps:

Step1:Determine the column (positional) value of each digit

Step2:Multiply the obtained column values by the digits in the corresponding columns

Step3: Calculate the sum of these products

Example:
$4706_8 = ?_{10}$

$= \overbrace{4 * 8^3} + \overbrace{7 * 8^2} + \overbrace{0 * 8^1} + \overbrace{6 * 8^0}$ —Common values multiplied by the corresponding digits

$= 4 * 512 + 7 * 64 + 0 * 8 + 6 * 1$

$= 2048 + 448 + 0 + 6$ — Sum of these products

$= (2502)_{10}$

★ 6- Converting a Decimal Number to a Number of Binary or Octal or Hexadecimal Base by using the Division-Remainder Method. To apply this method, we follow the following steps:

Step1: Divide the decimal number to be converted by the value of the new base.

Step2: Record the remainder from Step 1 as the rightmost the new base digit (least significant digit) of number.

Step3: Divide the quotient of the previous divide by the new base.

Step4: Record the remainder from Step 3 as the next digit (to the left) of the new base number.

Repeat Steps 3 and 4, recording remainders from right to left, until the quotient becomes zero in Step 3. Note that the last remainder thus new obtained will be the most significant digit (MSD) of the base number.

Example:
$952_{10} = ?_8$

$$
\begin{array}{rll}
8 & \underline{|\,952} & \\
8 & \underline{|\,119} & \text{remainder } 0 \\
8 & \underline{|\,14} & \text{remainder } 7 \\
8 & \underline{|\,1} & \text{remainder } 6 \\
 & 0 & \text{remainder } 1
\end{array}
$$

$952_{10} = 1670_8$

★ 7- Converting a Number from Some Bases to a Number of Another Base.
To apply this method, we follow the following steps:

Step1: Convert the original number to a decimal number (base 10).

Step2: Convert the decimal number so obtained to the new base number.

Example:
$545_6 = ?_4$

Step1: Convert base 6 to base 10.

Example:
$545_6 = 5 * 6^2 + 4 * 6^1 + 5 * 6^0$

$= 5 * 36 + 4 * 6 + 5 * 1$

$= 180 + 24 + 5$

$= (209)_{10}$

Step2: Convert $(209)_{10}$ to base 4.

$$
\begin{array}{r|l}
4 & 209 \\
4 & 52 \quad \text{remainder } 1 \\
4 & 13 \quad \text{remainder } 0 \\
4 & 3 \quad \text{remainder } 1 \\
 & 0 \quad \text{remainder } 3
\end{array}
$$

Hence $(209)_{10} = (3101)_4$

So $(545)_6 = (209)_{10} = (3101)_4$

★ 8- Converting a Binary Number to its Equivalent Octal Number, we need to follow the following steps:

Step1: Divide the digits into groups of three starting from the right.

Step2: Convert each group of three binary digits to one octal digit using the method of binary to decimal conversion.

Example:
$1101010_2 = ?_8$

Step1: Divide the digits into groups of three starting from the right.

<u>001</u> <u>101</u> <u>010</u>

Step1: Then convert each into one Octal digit.

$001_2 = 0 * 2^2 + 0 * 2^1 + 1 * 2^0 = 1$

$101_2 = 1 * 2^2 + 0 * 2^1 + 1 * 2^0 = 5$

$010_2 = 0 * 2^2 + 1 * 2^1 + 0 * 2^0 = 2$

Hence $(1101010)_2 = (152)_8$

★ 9- Converting an Octal Number to its Equivalent Binary Number, we need to follow the following steps:

Step1: Convert each octal digit to a 3 digit binary number (the octal digits may be treated as decimal for this conversion).

Step2: Merge all the resulting binary groups (of 3 digits each) into a single binary number.

Example:
$562_8 = ?_2$

Step1: Convert each octal digit to a 3 digit binary number .

$5_8 = 101_2 \qquad 6_8 = 110_2 \qquad 2_8 = 010_2$

Step1: Combine the binary groups .

$(562)_8 = 101110010_2$

Hence $(562)_8 = (101110010)_2$

★ 10- Converting a Binary Number to its Equivalent Hexadecimal Number by applying the following steps:

Step1: Divide the binary digits into groups of four starting from the right.

Step2: Merge each group of four binary digits to one hexadecimal digit.

Example:
$111101_2 = ?_{16}$

Step1: Divide the binary digits into groups of four starting from the right.

0011 1101

Step2: Convert each group into a hexadecimal digit.

$$0011_2 = 0 * 2^3 + 0 * 2^2 + 1 * 2^1 + 1 * 2^0 = 3_{10} = 3_{16}$$

$$1101_2 = 1 * 2^3 + 1 * 2^2 + 0 * 2^1 + 1 * 2^0 = 13_{10} = D_{16}$$

Hence $(11110)_2 = (3D)_{16}$

★ 11- Converting a Hexadecimal Number to its Equivalent Binary Number by applying the following steps:

Step1: Convert the decimal equivalent of each hexadecimal digit to a 4 digit binary number.

Step2: Combine all the resulting binary groups (of 4 digits each) in a single binary number.

Example:
$2AB_{16} = ?_2$

Step1: Convert each hexadecimal digit to a 4 digit binary number.

$$2_{16} = 2_{10} = 0010_2$$

$$A_{16} = 10_{10} = 1010_2$$

$$B_{16} = 11_{10} = 1011_2$$

Step2: Combine the binary groups.

$$2AB_{16} = 001010101011_2$$

Table 1: Relationship between Hexadecimal, Octal, Decimal, and Binary

D	H	B	O
0	0	0000	0
1	1	0001	1
2	2	0010	2
3	3	0011	3
4	4	0100	4
5	5	0101	5
6	6	0110	6
7	7	0111	7
8	8	1000	10
9	9	1001	11
10	**A**	1010	12
11	**B**	1011	13
12	**C**	1100	14
13	**D**	1101	15
14	**E**	1110	16
15	**F**	1111	17

Hence $(2AB)_{16} = (001010101011)_2$

★ 12- Notice that each hexadecimal digit represents a group of four binary digit. It is important to remember that digit A through F are equivalent to the decimal value 10 through 15.