Logical Design Lectures

Produced By Dr. Faris Llwaah

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Lecture 2 – Signed Binary Number

- Positive integers (including zero) can be represented as un-signed numbers.
- To represent negative integer numbers, we need a notation to represent the negative values.
- In ordinary arithmetic, a negative number is indicated by a MINUS sign –ve, and PLUS sign for a positive number +ve.
- In ordinary arithmetic, a negative number is indicated by a MINUS sign –ve, and PLUS sign for a positive number +ve.
- Due to a computer hardware limitations, computer must represent every thing in the binary digits form.
- It is customary to represent the sign with a bit placed in the left position of the number.
- It is convention to specify the sign bit "0" for positive and "1" for negative number.
- The **PROBLEM** with signed integers (-45, +27, -99) is the sign. How do we encode the sign?

Representation of negative numbers

There are three forms in which signed integer numbers can be represented in Binary.

1- **Sign−magnitude (S&M).** Negates number by changing its sign bit.
2- **1's complement form.** Negates number by taking its 1's complement.
3- **2's complement form.** Negates number by taking its 2's complement.

Sign−magnitude (S&M).

- The left-most in a signed binary number is the sign bit, which tell us if the number is positive or negative. "0" for +ve, and "1" for –ve.
- Signed magnitude (SM) is a method for encoding signed integers.
- However the most significant bit is used to represent the sign "1" is used for a − (negative sign), a "0" for a + (positive). The following format is a SM number in 8 bits.

S mmmmmmm

Where S is the sign bit and the other 7 bits represent the magnitude.
Note— For positive numbers, the result is the same as the unsigned binary representation.

Example:

- Magnitude is magnitude, *does not change with sign*

S	Magnitude (Binary)

$(+3)_{10} \Rightarrow (0\,011)_2$

$(-3)_{10} \Rightarrow (1\,011)_2$

Sign Magnitude

- Can't include the *sign bit* in 'Addition'

$$\begin{array}{rl} & 0\,0\,1\,1 \Rightarrow (+3)_{10} \\ + & 1\,0\,1\,1 \Rightarrow (-3)_{10} \\ \hline & 1\,1\,1\,0 \Rightarrow (-6)_{10} \end{array}$$

Examples:

$-5 = 1\ 0000101_2 = 85_{16}$
$+5 = 0\ 0000101_2 = 05_{16}$
$+127 = 0\ 1111111_2 = 7F_{16}$
$-127 = 1\ 1111111_2 = FF_{16}$
$+0 = 0\ 0000000_2 = 00_{16}$
$-0 = 1\ 0000000_2 = 80_{16}$

The 1's complement.

To encode a negative number, get the binary representation of its magnitude then COMPLEMENT each bit. Complementing each bit mean that 1's are replaced with 0's, 0's are replaced with 1's.

Example:

What is -5 in 1's complement, in the form of 8 bits?
Now complement each bit: $11111010 = FA_{16}$
Note: Positive numbers in 1's complement are simply their binary representation.

Examples:

$-5 = 11111010_2 = FA_{16}$
$+5 = 00000101_2 = 05_{16}$
$+127 = 0111111_2 = 7F_{16}$
$-127 = 10000000_2 = 80_{16}$
$+0 = 00000000_2 = 00_{16}$
$-0 = 11111111_2 = 80_{16}$

Note: For the 8 bits, we can represent minimum value is -127 and the maximum is +127.

- Still we have a problem which is there are two ways of representing 0. However, addition of $K + (-K)$ gives Zero.

  Example:

  $-5 + 5 = FA_{16} + 05_{16} = FF_{16} = -0!!!!$

  Also, $K + 0$ works only if we use $+0$, because it doesn't work if we use -0.

  $5 + (+0) = 05_{16} + 00_{16} = 05_{16} = 5 \quad (OK)$
  $5 + (-0) = 05_{16} + FF_{16} = 04_{16} = 4 \quad (Wrong)$

The 2's complement.

To encode a negative number, get the binary representation of its magnitude then COMPLEMENT each bit. Then add 1 to the 1's complement (It means get the 1's complement and ADD 1 to it).

Example:

What is -5 in 2's complement, in the form of 8 bits?
Now complement each bit: $11111010 = FA_{16}$ and then ADD 1 to the FA.
$FA_{16} + 1 = FB_{16}$
Where FB is a 8 bits 2's complement of the -5.

Note: Positive numbers in 2's complement are simply their binary representation.

Examples:

$-5 = 11111011_2 = FB_{16}$
$+5 = 00000101_2 = 05_{16}$
$+127 = 0111111_2 = 7F_{16}$
$-127 = 10000001_2 = 81_{16}$
$-128 = 10000000_2 = 80_{16}$ the extended range
$+0 = 00000000_2 = 00_{16}$
$-0 = 00000000_2 = 00_{16}$ we have Only one Zero

Note: For the 8 bits, we can represent the signed integer as a minimum value is -128 and the maximum is +127.
For N bits, we can represent the signed integers as follows:

$$-2^{(N-1)} \quad to \quad +2^{(N-1)} - 1$$

This method has none of the drawbacks of signed magnitude or one's complement. There is only on zero, and $K + (-K) = 0$.

Example:

$-5 + 5 = FB_{16} + 05_{16} = 00_{16} = 0$ (Correct)

Example:

Express the decimal number (-39) as the 8 bits number in the following forms:
1- Sing-magnitude.
2- 1's complement.
3- 2's complement.
Solution:
Find out the Binary number of $(+39)$
$(+39)_{10} = 00100111$

1- In the sign magnitude form, (-39) can be produced by changing the sign bit only to "1" and leave the magnitude bits as they are. So (-39) will be 10100111.

2- In the 1's complement, change each "1" to "0" and each "0" to "1". So (-39) will be 11011000

3- In the 2's complement, get 1's complement of (-39) and add "1" to it. So (-39) will be 11011001

Decimal number of Signed number.

1- Sign-Magnitude (S&M): Decimal values of $+ve$ and $-ve$ number in the SM form can be determined by getting the summation of the weights in the magnitude part. To calculate the weights, we consider those bits that have ones only "1", and ignoring those bits that have zeros "0".

Example:

Define the decimal value in sign magnitude form of the following binary number: 10010011

The seven magnitude bits and their weights as follows:

0010011

$$\underbrace{64*0}_{0}+\underbrace{32*0}_{0}+\underbrace{16*1}_{16}+\underbrace{8*0}_{0}+\underbrace{4*0}_{0}+\underbrace{2*1}_{2}+\underbrace{1*1}_{1}=+19$$

If the sign bit is "1", so the binary number of $(-19) = 1010011$

2- 1's complement:

- Decimal values of $+ve$ numbers in 1's complement form are determined by summing the weights in all bit position that have Ones and ignoring those bit positions that have zeros.

- Decimal values of $-ve$ numbers are determined by assigning a $-ve$ values to the weight of the sign bit, then summing all weights where there are Ones, and adding 1 to the result.

Example:

Determine the decimal values of the following signed binary numbers using 1's complement:
A- 00010111

B- 11101000

A- $\underbrace{-128*0}_{0}+\underbrace{64*0}_{0}+\underbrace{32*0}_{0}+\underbrace{16*1}_{16}+\underbrace{8*0}_{0}+\underbrace{4*1}_{4}+\underbrace{2*1}_{2}+\underbrace{1*1}_{1}=$
$= 16+4+2+1 = +23$

B- $\underbrace{-128*1}_{-128}+\underbrace{64*1}_{64}+\underbrace{32*1}_{32}+\underbrace{16*0}_{0}+\underbrace{8*1}_{8}+\underbrace{4*0}_{0}+\underbrace{2*0}_{0}+\underbrace{1*0}_{0}=$
$= -128+64+32+8 = -24$
Adding "1" to the result, the final decimal number is -23

2- 2's complement:
Decimal values of $+ve$ and $-ve$ numbers in 2's complement form are determined by getting the summation the weight in bits position where there are oncs and ignoring those bit positions having zeros.

Example:

Determine the decimal value of the following singed numbers using 2's complement:

A- 01010110
B- 10101010

A- $\underbrace{-128*0}_{0}+\underbrace{64*1}_{64}+\underbrace{32*0}_{0}+\underbrace{16*1}_{16}+\underbrace{8*0}_{0}+\underbrace{4*1}_{4}+\underbrace{2*1}_{2}+\underbrace{1*0}_{0}=$
$= 64+16+4+2 = +86$

A- $\underbrace{-128*1}_{-128}+\underbrace{64*0}_{0}+\underbrace{32*1}_{32}+\underbrace{16*0}_{0}+\underbrace{8*1}_{8}+\underbrace{4*0}_{0}+\underbrace{2*1}_{2}+\underbrace{1*0}_{0}=$
$= -128+32+8+2 = -86$

The range of unsigned and signed number.

- The 8 bit is used to illustrate a number because the storage unit in the computer is known as a Byte.

- The maximum decimal number that can be represented with 1 Byte is 255 or 11111111. An 8-bit word greatly restricts the range of numbers that can be accommodated. The maximum number of values is 256 or 0 through 255.

- The maximum decimal number that can be represented with 2 Bytes (Word) is 65536. The represented value could be 0, 1, 2, 3, through 65535.

- The maximum decimal number that can be represented with 4 Bytes (Double word) is $4.295 * 10^9$

- The formula to find the numbers of different combinations of bits as follows:

  Total value $= 2^n$

  A- **Unsigned number:**

- For 8 bits

  $Minimum: 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0$

  $Maximum: 1\ \ 1\ \ 1\ \ 1\ \ 1\ \ 1\ \ 1\ \ 1$

  $Hence\ \ 2^8 = 256\ \ From\ \ 0 - -255$

  B- **Signed number- Signed & Magnitude (SM):**

- For $+ve$ number

  $Minimum: \overbrace{0}^{S}\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0 = +0$

  $Maximum: \overbrace{0}^{S}\ \ 1\ \ 1\ \ 1\ \ 1\ \ 1\ \ 1\ \ 1 = +127$

- For $-ve$ number

$Minimum: \overbrace{1}^{S}\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0 = -0$

$Maximum: \overbrace{1}^{S}\ \ 1\ \ 1\ \ 1\ \ 1\ \ 1\ \ 1\ \ 1 = -127$

Range ==> -127 —> +127

C- **Signed number- 1's complement:**

- For $+ve$ number

$Minimum: \mathbf{0}\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0 = +0$

$Maximum: \mathbf{0}\ \ 1\ \ 1\ \ 1\ \ 1\ \ 1\ \ 1\ \ 1 = +127$

- For $-ve$ number

$Minimum: \mathbf{1}\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0 = -127$

$Maximum: \mathbf{1}\ \ 1\ \ 1\ \ 1\ \ 1\ \ 1\ \ 1\ \ 1 = -0$

Range ==> -127 —> +127

D- **Signed number- 2's complement:**

- For $+ve$ number

$Minimum: \mathbf{0}\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0 = 0$