Logical Design Lectures

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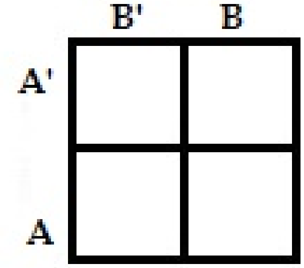
Lecture 4 – Karnaugh Map

- Karnaugh map is a graphical method that is used to simplify a logical equation or to convert a truth table to its corresponding logic circuit in a simple, orderly process.
- By using Karnaugh map technique, we can reduce the Boolean expression containing any number of variables, such as 2-variable Boolean expression, 3-variable Boolean expression, 4-variable Boolean expression and even 7-variable Boolean expressions, which are complex to solve by using regular Boolean theorems and laws. In our study we consider boxes/ squares shapes called " cells" as follows:
- (2) variables K-map contains $2^2 = 4$ cells. It will look like the following image



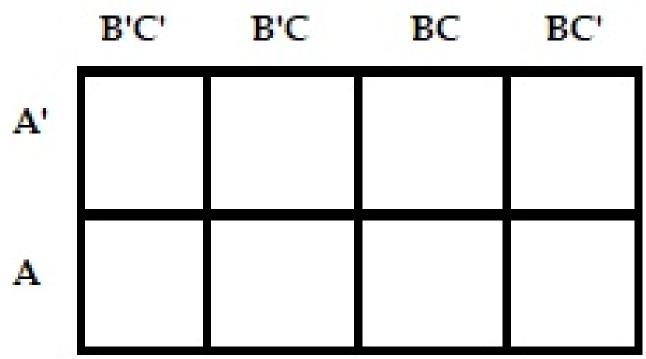
- The possible min terms with 2 variables ($AandB$) are $A.B$, $A.\overline{B}$, $\overline{A}.B$ and $\overline{A}.\overline{B}$. The conjunctions of the variables (A, B) and ($\overline{A}, B$) are represented in the cells of the top row and ($A, \overline{B}$) and ($\overline{A}, \overline{B}$) in cells of the bottom row. The following table shows the positions of all the possible outputs of 2-variable Boolean function on a K-map.

A	B	Possible Outputs	Location on K-map
0	0	A'B'	0
0	1	A'B	1
1	0	AB'	2
1	1	AB	3

- A general representation of a 2 variable K-map plot is shown below.

A \ B	0	1
0	A'B' 0	A'B 1
1	AB' 2	AB 3

- (3) variables K-map contains $2^3 = 8$ cells. It will look like the following image



- For a 3-variable Boolean function, there is a possibility of 8 output min terms. The general representation of all the min terms using 3-variables is shown below.

A	B	C	Output Function	Location on K-map
0	0	0	A'B'C'	0
0	0	1	A'B'C	1
0	1	0	A'BC'	2
0	1	1	A'BC	3
1	0	0	AB'C'	4
1	0	1	AB'C	5
1	1	0	ABC'	6
1	1	1	ABC	7

- A typical plot of a 3-variable K-map is shown below. It can be observed that the positions of columns 10 and 11 are interchanged so that there is only change in one variable across adjacent cells. This modification will allow in minimizing the logic.

A \ BC	00	01	11	10
0	A’B’C’ [0]	A’B’C [1]	A’BC [3]	A’BC’ [2]
1	AB’C’ [4]	AB’C [5]	ABC [7]	ABC’ [6]

- (4) variables K-map contains $2^4 = 16$ cells. It will look like the following image

	C'D'	C'D	CD	CD'
A'B'				
A'B				
AB				
AB'				

- There are 16 possible min terms in case of a 4-variable Boolean function. The general representation of min-terms using 4 variables is shown below.

A	B	C	D	Output function	K-map location
0	0	0	0	A’ B’ C’ D’	0
0	0	0	1	A’ B’ C’ D	1
0	0	1	0	A’ B’ C D’	2
0	0	1	1	A’ B’ C D	3
0	1	0	0	A’ B C’ D’	4
0	1	0	1	A’ B C’ D	5
0	1	1	0	A’ B C D’	6
0	1	1	1	A’ B C D	7
1	0	0	0	A B’ C’ D’	8
1	0	0	1	A B’ C’ D	9
1	0	1	0	A B’ C D’	10
1	0	1	1	A B’ C D	11
1	1	0	0	A B C’ D’	12
1	1	0	1	A B C’ D	13
1	1	1	0	A B C D’	14
1	1	1	1	A B C D	15

- A typical 4-variable K-map plot is shown below. It can be observed that both the columns and rows of 10 and 11 are interchanged.

AB \ CD	00	01	11	10
00	0 $A'B'C'D'$	1 $A'B'C'D$	3 $A'B'CD$	2 $A'B'CD'$
01	4 $A'BC'D'$	5 $A'BC'D$	7 $A'BCD$	6 $A'BCD'$
11	12 $ABC'D'$	13 $ABC'D$	15 $ABCD$	14 $ABCD'$
10	8 $AB'C'D'$	9 $AB'C'D$	11 $AB'CD$	10 $AB'CD'$

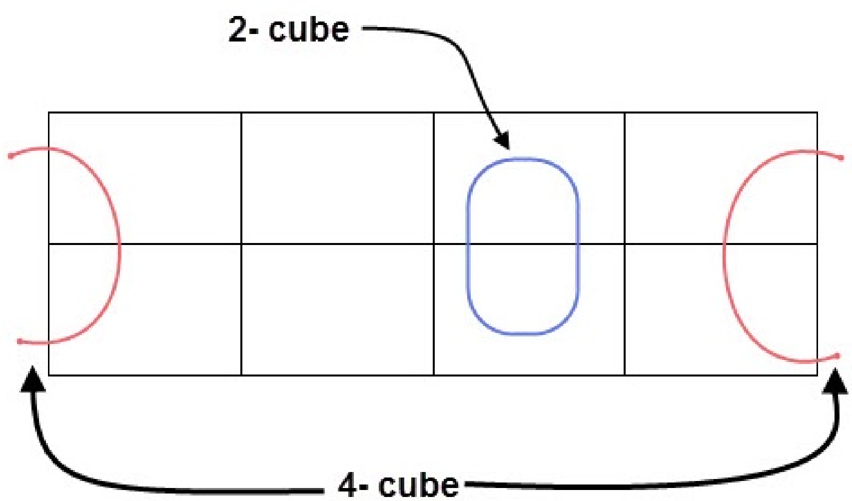
■ Minimization with Karnaugh Maps and advantages of K-map

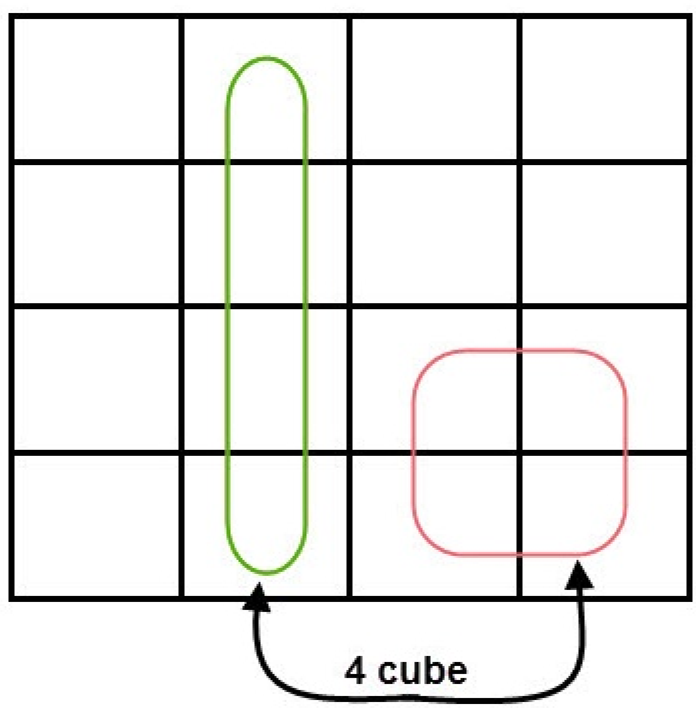
- K-maps are used to convert the truth table of a Boolean equation into minimized SOP form.
- Easy and simple basic rules for the simplification.
- The K-map method is faster and more efficient than other simplification techniques of Boolean algebra.
- All rows in the K-map are represented by using a square shaped cells, in which each square in that will represent a min-term.
- It is easy to convert a truth table to k-map and k-map to Sum of Products form equation.

■ Grouping of K-map variables

- There are some rules to follow while we are grouping the variables in K-maps. They are as follows:
- The square that contains "1" should be taken in simplifying, at least once.
- Group shouldn't include any zeros (0).

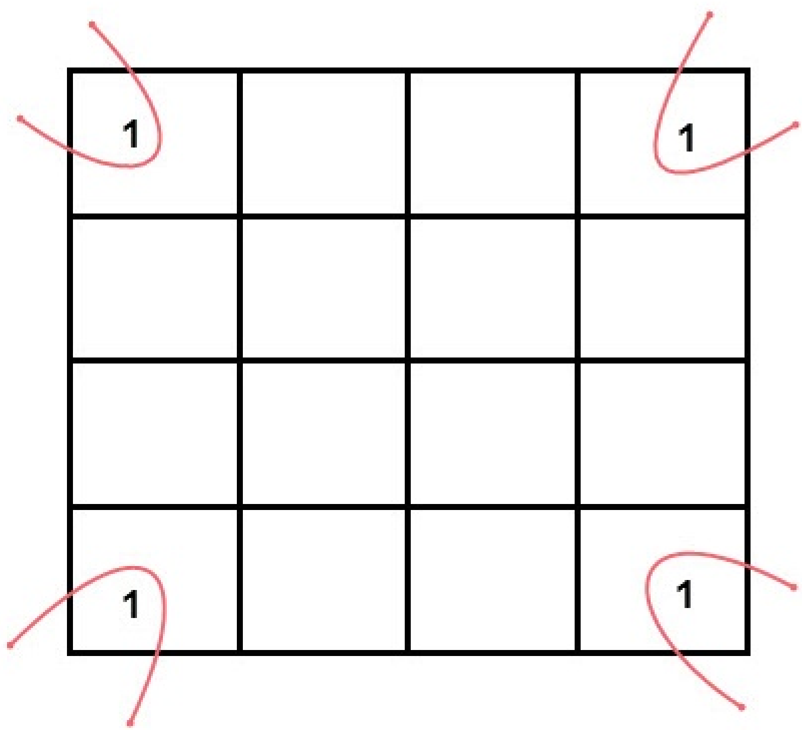
- A group should be the as large as possible.
- Groups can be horizontal or vertical. Grouping of variables in diagonal manner is not allowed.

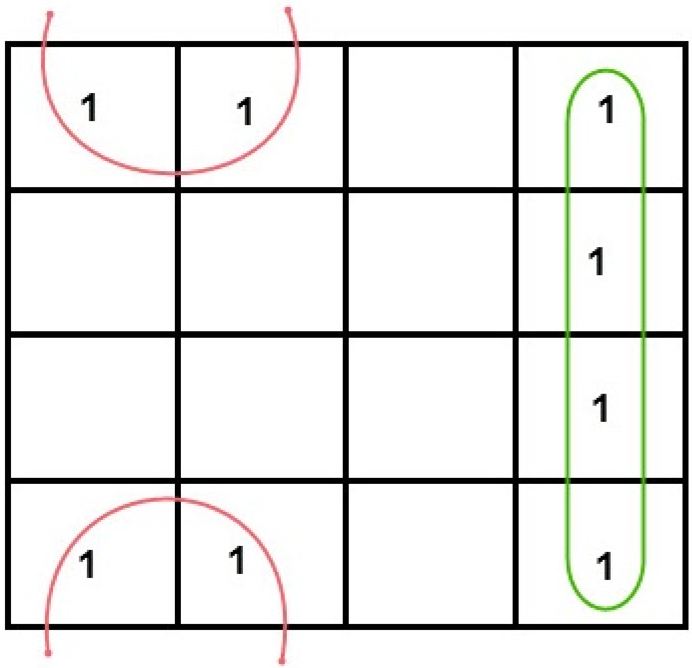


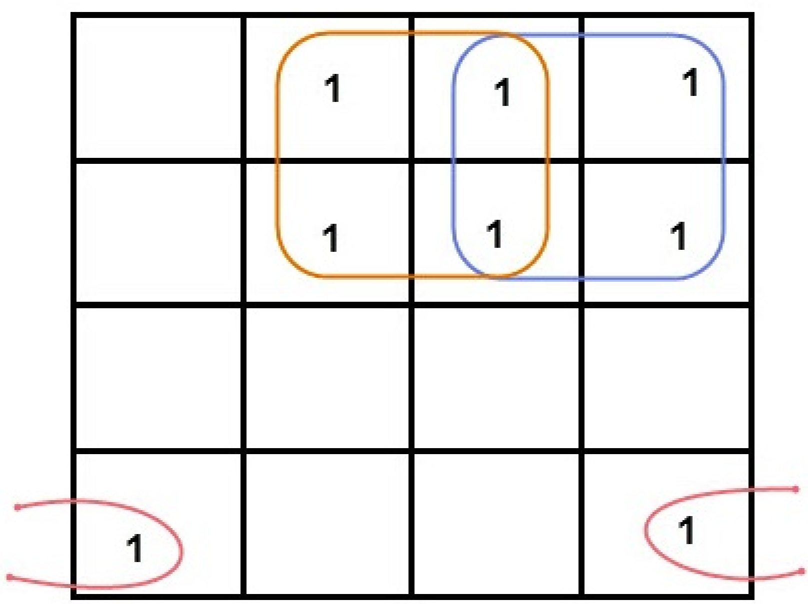


- If the square containing '1' has no possibility to be placed in a group, then it should be added to the final expression.

- Groups can overlap.
- The number of squares in a group must be equal to powers of 2, such as 1, 2, 4, 8 etc.
- Groups can wrap around. As the K-map is considered as spherical or folded, the squares at the corners (which are at the end of the column or row) should be considered as they adjacent squares.
- The grouping of K-map variables can be done in many ways, so the obtained simplified equation need not to be unique always.
- The Boolean equation must be in must be in canonical form, in order to draw a K-map.
- For example see the following K-Map:







Example:

Find an optimization equation of the following K-Map:

B_3B_2 \ B_1B_0	00	01	11	10
00	0	0	0	0
01	0	0	0	0
11	1	1	1	1
10	1	1	1	1

Solution:

Equation for $G = B_3$

Example:

Find an optimization equation of the following K-Map:

$B_3B_2 \backslash B_1B_0$	00	01	11	10
00	0	0	0	0
01	1	1	1	1
11	0	0	0	0
10	1	1	1	1

Solution:

Equation for $G = \overline{B3}B2 + B3\overline{B2} = B3 \oplus B2$

Exercise:

Simplify the following expression using 1- Boolean Algebra 2- K-Map.

$Y = \overline{A}B + A\overline{B} + AB$

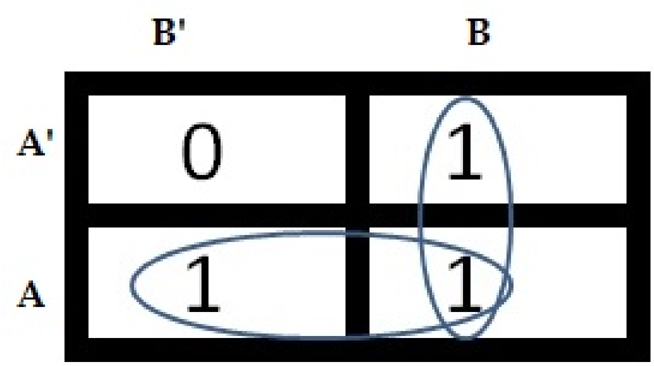
1- Boolean Algebra:

$Y = \overline{A}B + A\overline{B} + AB + AB$... *Applying* $(A + A = A)$ *Rule* . So we have added AB.

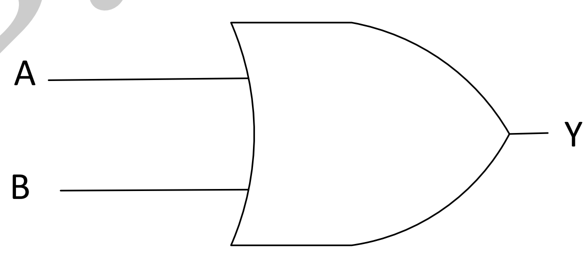
$Y = A(B + \overline{B}) + B(\overline{A} + A)$

$Y = A + B$

2- K-Map:



$Y = A + B$, It represents OR Gate



Exercise:

Simplify the following function using Karnaugh map:

$Y = \sum_1 (0, 1, 4, 5, 8, 9, 10, 11, 14, 15)$

	C'D	C'D	CD	CD'
A'B'	0: 1	1: 1	3:	2:
A'B	4: 1	5: 1	7:	6:
AB	12:	13:	15: 1	14: 1
AB'	8: 1	9: 1	11: 1	10: 1

$$Y = \overline{A}\ \overline{C} + AC + A\overline{B}$$