Logical Design Lectures

Produced By Dr. Faris Llwaah

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Lecture 3 – Boolean Algebra

- Boolean Algebra is a part of the Combinational Logic topics. Different from the Sequential logic topics (can store information).

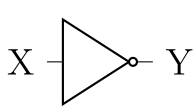
- Learning theorems of Boolean algebra allows you to design logic functions.

- Learning theorems of Boolean algebra allows you to know how to combine different logic gates.

- Learning theorems of Boolean algebra allows you to simplify or optimize on the complex operations.

■ Basic Logic Gates

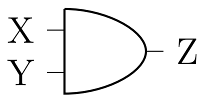
- **NOT- Gate Inverter**



$$Y = \overline{X}$$

X	Y
0	1
1	0

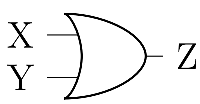
- **AND- Gate Multiplication**



$$Z = X \bullet Y$$

X	Y	Z
0	0	0
0	1	0
1	0	0
1	1	1

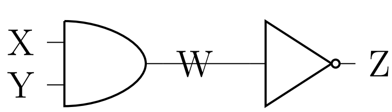
- **OR- Gate**



$$Z = X + Y$$

X	Y	Z
0	0	0
0	1	1
1	0	1
1	1	1

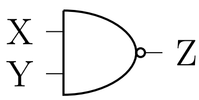
- **NAND- (NOT+AND) Gate**



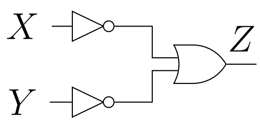
$$Z = \overline{W} = \overline{(X \bullet Y)}$$

X	**Y**	**W**	**Z**
0	0	0	1
0	1	0	1
1	0	0	1
1	1	1	0

- **NAND- Gate**



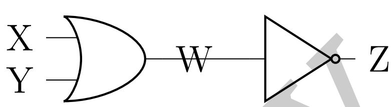
$$Z = \overline{(X \bullet Y)}$$



$$Z = \overline{X} + \overline{Y}$$

X	**Y**	**∼X**	**∼Y**	**Z**
0	0	1	1	1
0	1	1	0	1
1	0	0	1	1
1	1	0	0	1

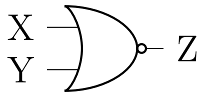
- **NOR- (NOT+OR) Gate**



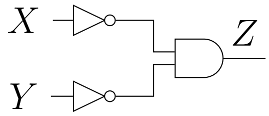
$$Z = \overline{W} = \overline{(X + Y)}$$

X	**Y**	**W**	**Z**
0	0	0	1
0	1	1	0
1	0	1	0
1	1	1	0

- **NOR- Gate**



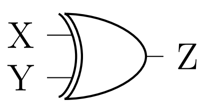
$$Z = \overline{(X+Y)}$$



$$Z = \overline{X} \bullet \overline{Y}$$

X	**Y**	**∼X**	**∼Y**	**Z**
0	0	1	1	1
0	1	1	0	0
1	0	0	1	0
1	1	0	0	0

- **XOR- Exclusive OR Gate**

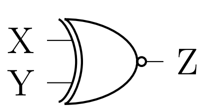


$$Z = X \bullet \overline{Y} + \overline{X} \bullet Y = X \oplus Y$$

- The output of an XOR gate is true (1) only when exactly one of its inputs is true (1). If both of an XOR gate's inputs are false (0). Or if both its inputs are true (1), then the output of the XOR is false (0).

X	**Y**	**Z**
0	0	0
0	1	1
1	0	1
1	1	0

- **XNOR- Exclusive OR Gate**



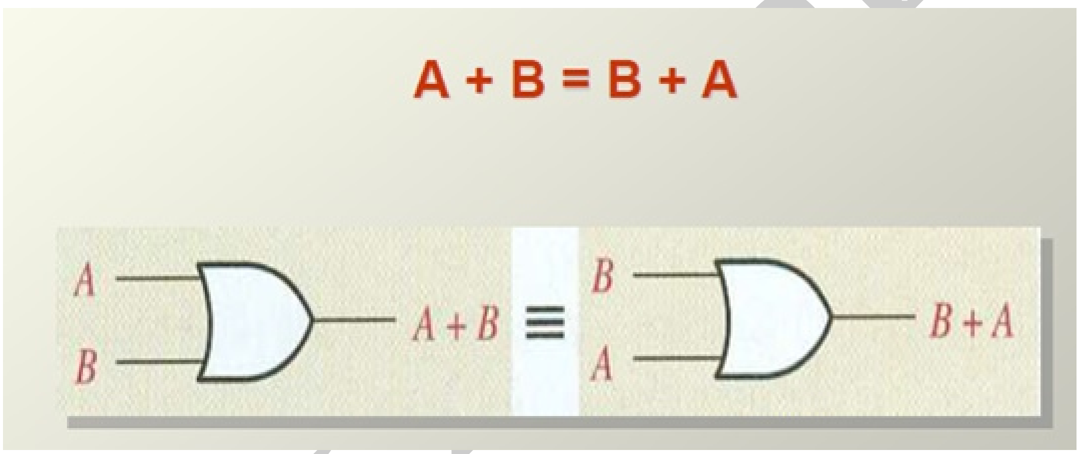
$$Z = \overline{X} \bullet \overline{Y} + X \bullet Y = \overline{X \oplus Y}$$

- The output of an XNOR gate is true (1) only when all of its inputs are true (1) or when all of its inputs are false (0). If some of its inputs are true (1) and others are false (0), then the output of the XNOR is false (0).

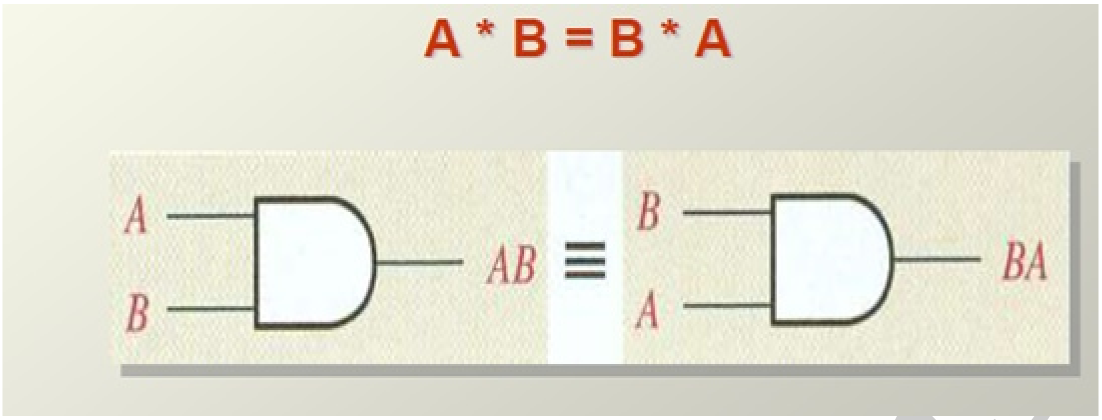
X	**Y**	**Z**
0	0	1
0	1	0
1	0	0
1	1	1

■ Laws of Boolean Algebra

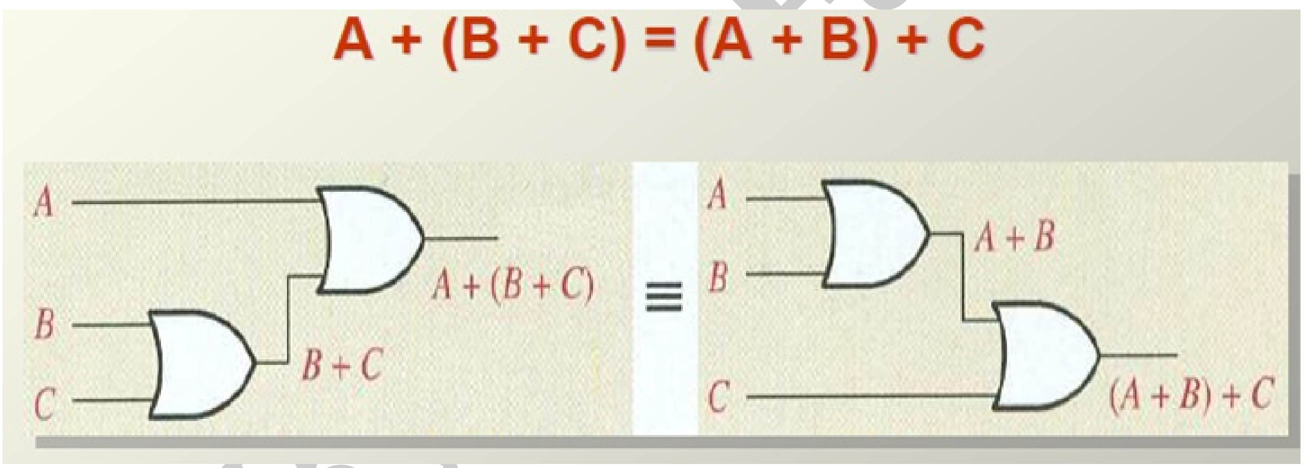
- Commutative Law of Addition:



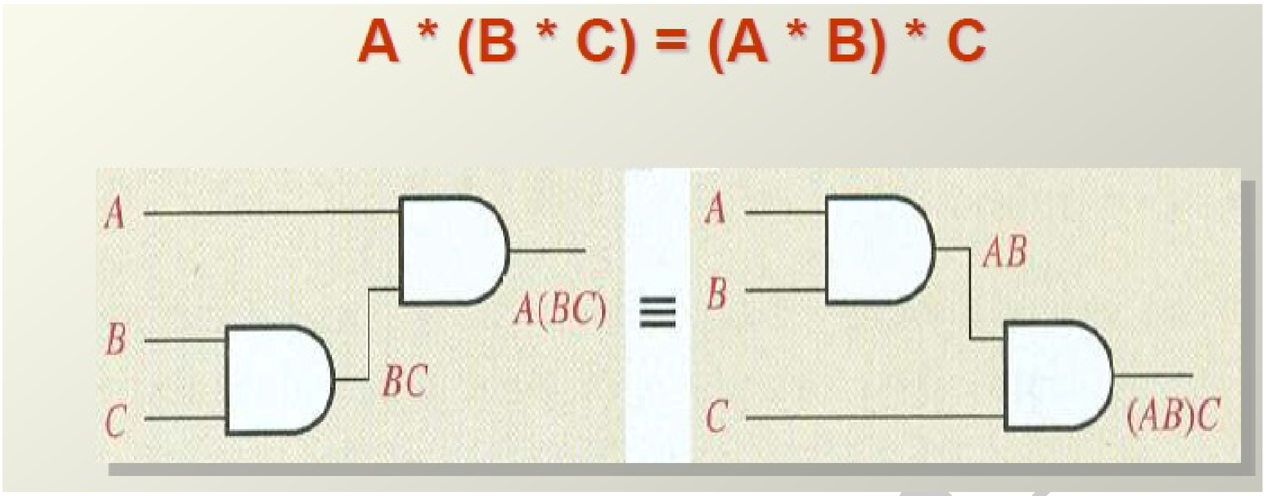
- Commutative Law of Multiplication:



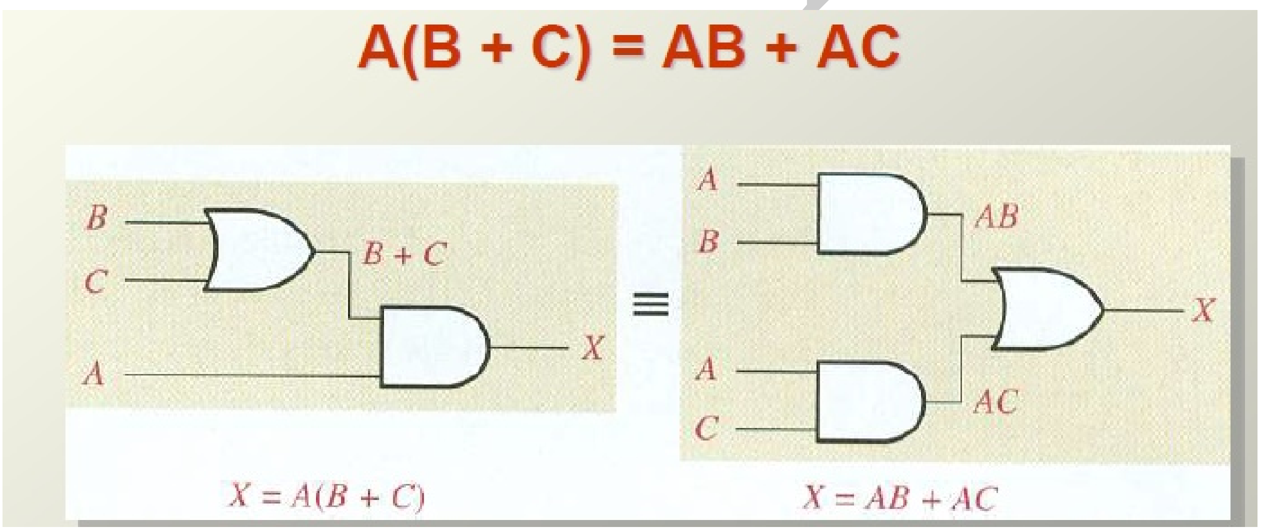
- Associative Law of Addition:



• Associative Law of Multiplication:



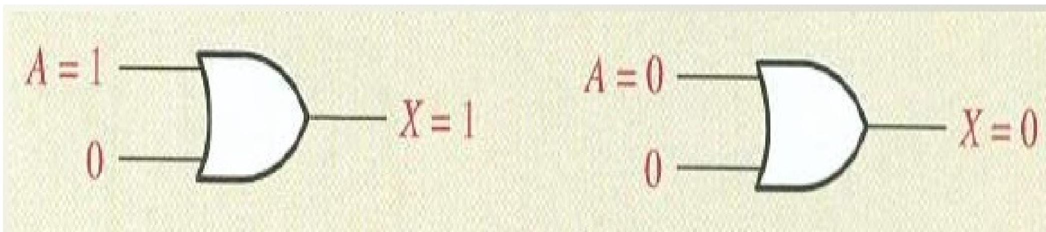
• Distributive Law:



■ Rules of Boolean Algebra

- Rule 1 Identity (ADDITION):

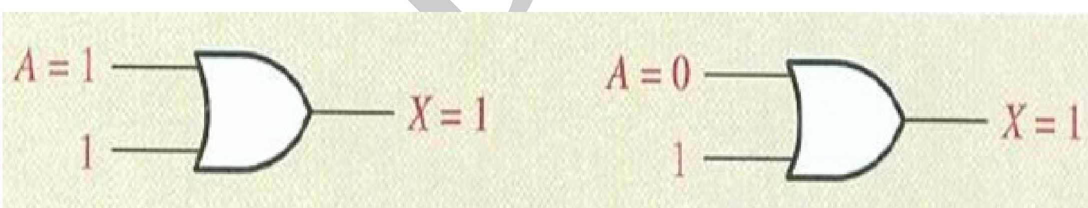
$X = A + 0 = A$



A	B	X
0	0	0
0	1	1
1	0	1
1	1	1

- Rule 2 NULL (ADDITION):

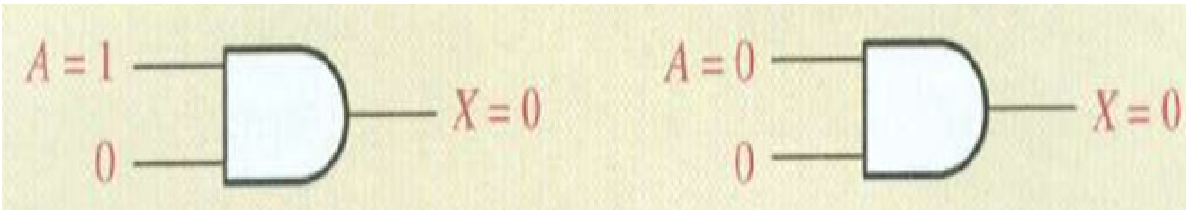
$X = A + 1 = 1$



A	B	X
0	0	0
0	1	1
1	0	1
1	1	1

- Rule 3 NULL (MULTIPLICATION):

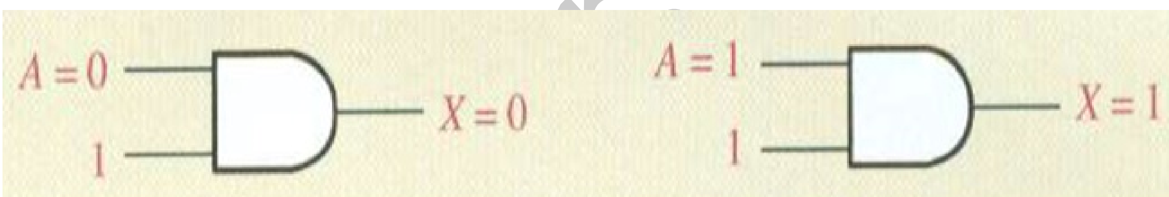
  $X = A * 0 = 0$



A	B	X
0	0	0
0	1	0
1	0	0
1	1	1

- Rule 4 Identity (MULTIPLICATION):

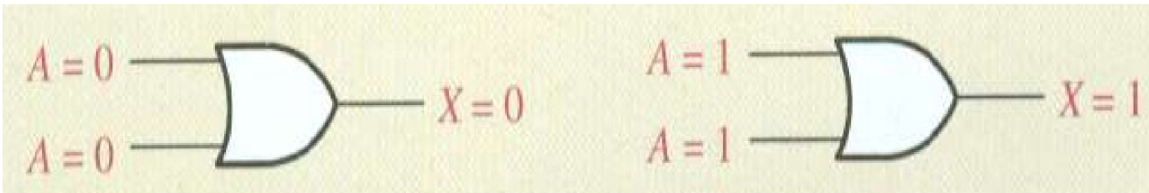
  $X = A * 1 = A$



A	B	X
0	0	0
0	1	0
1	0	0
1	1	1

- Rule 5 Idempotent (ADDITION):

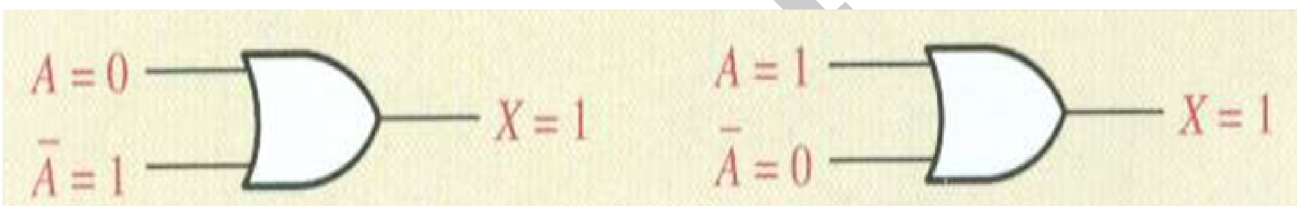
  $X = A + A = A$



A	B	X
0	0	0
1	1	1

- Rule 6 Complementary (ADDITION):

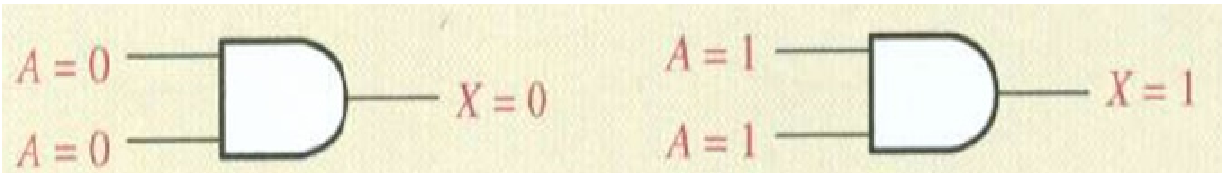
  $X = A + \overline{A} = 1$



A	B	X
0	0	0
0	1	1
1	0	1
1	1	1

- Rule 7 Idempotent (MULTIPLICATION):

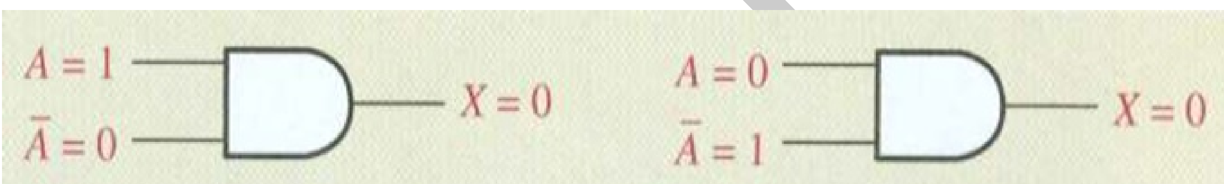
$X = A * A = A$



A	**A**	**X**
0	0	0
1	1	1

- Rule 8 Complementary (MULTIPLICATION):

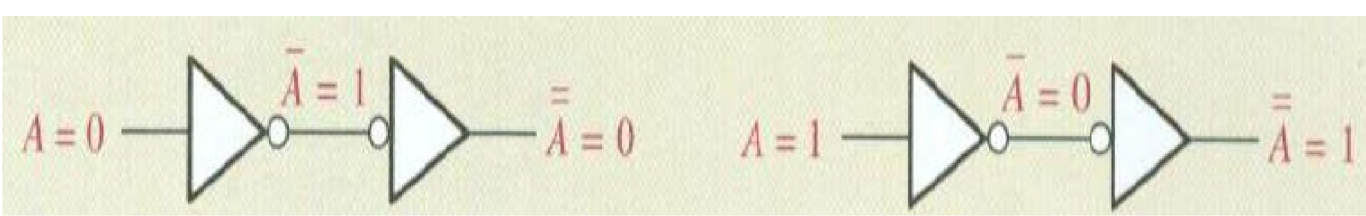
$X = A * \overline{A} = 0$



A	**A**	**X**
0	1	0
1	0	0

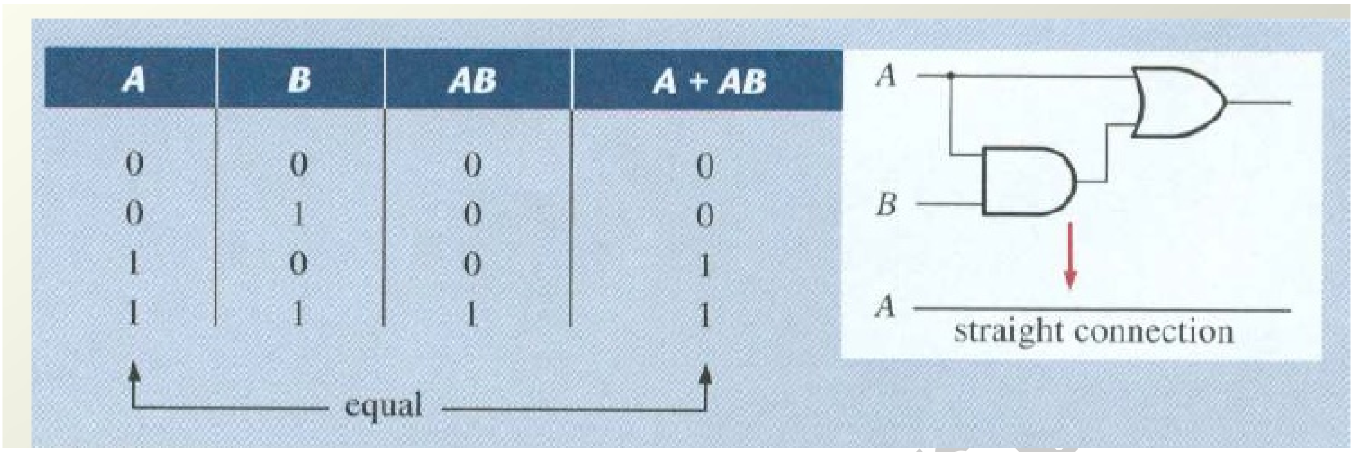
- Rule 9 Involution:

$A = \overline{\overline{A}}$



- Rule 10 OR Absorption:

$A + (A \bullet B) = A$



A	B	AB	A + AB
0	0	0	0
0	1	0	0
1	0	0	1
1	1	1	1

The proof:
Left side = A + AB

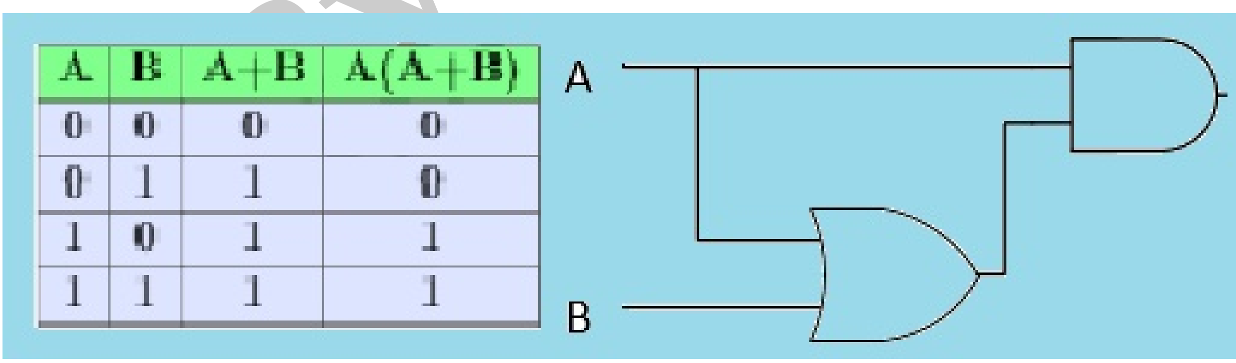
= A (1+B)

= $A \bullet 1$

= A

- Rule 11 AND Absorption:

$A \bullet (A + B) = A$



A	B	A+B	A(A+B)
0	0	0	0
0	1	1	0
1	0	1	1
1	1	1	1

The proof:
Left side = A(A+B)