$$= A \bullet A + AB$$

$$= A + AB$$

$$= A(1 + B)$$

$$= A \bullet 1$$

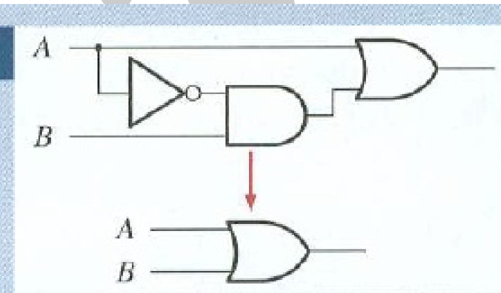
$$= A$$

- Rule 12 Redundancy:

$$A + \overline{A}B = (A + B)$$

A	B	$\overline{A}B$	$A + \overline{A}B$	$A + B$
0	0	0	0	0
0	1	1	1	1
1	0	0	1	1
1	1	0	1	1

($A + \overline{A}B$ and $A + B$: equal)



The proof:
Left side $= A + \overline{A}B$

$$= A + AB + \overline{A}B$$

$$= A + B(A + \overline{A})$$

$$= A + B \bullet 1$$

$$= A + B$$

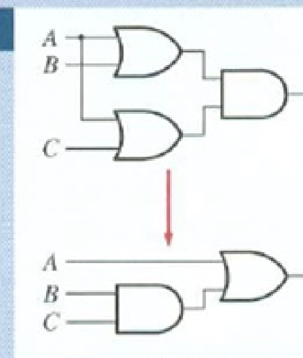
= Right side

- Rule 13:

$(A + B)(A + C) = A + BC$

A	B	C	A + B	A + C	(A + B)(A + C)	BC	A + BC
0	0	0	0	0	0	0	0
0	0	1	0	1	0	0	0
0	1	0	1	0	0	0	0
0	1	1	1	1	1	1	1
1	0	0	1	1	1	0	1
1	0	1	1	1	1	0	1
1	1	0	1	1	1	0	1
1	1	1	1	1	1	1	1

equal



The proof:
Left side $= (A + B)(A + C)$

$$= AA + AC + AB + BC$$

$$= A + AC + AB + BC$$

$$= A(1 + C + B) + BC$$

$$= A \bullet 1 + BC$$

$$= A + BC$$

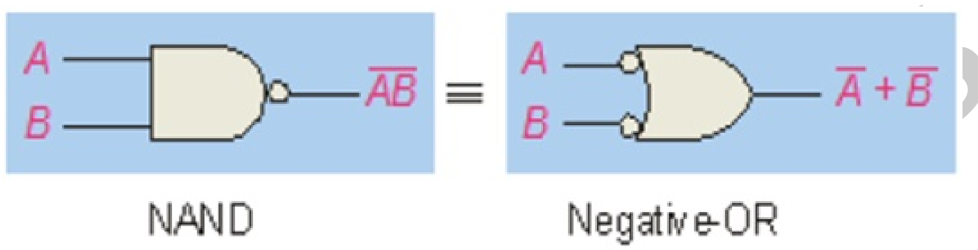
$$= \text{Right side}$$

■ DeMorgan's Theorem (First Theorem)

- The complement of a product of variables is equal to the sum of complemented variables.

$$\overline{AB} = \overline{A} + \overline{B}$$

- Applying DeMorgan's First Theorem to gates:



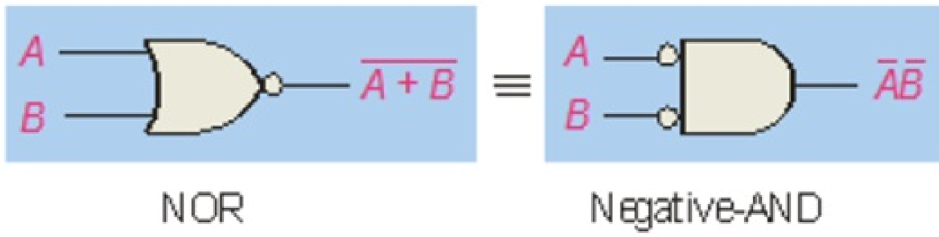
A	B	∼A	∼B	∼AB	∼A+∼B
0	0	1	1	1	1
0	1	1	0	1	1
1	0	0	1	1	1
1	1	0	0	0	0

■ DeMorgan's Theorem (Second Theorem)

- The complement of a sum of variables is equal to the product of complemented variables.

$$\overline{A+B} = \overline{A} \bullet \overline{B}$$

- Applying DeMorgan's Second Theorem to gates:



A	B	~A	~B	~(A+B)	~A.~B
0	0	1	1	1	1
0	1	1	0	0	0
1	0	0	1	0	0
1	1	0	0	0	0

Example:

Apply DeMorgan's theorem to simplify the following expression.

$$X = \overline{\overline{C} + D}$$

To apply DeMorgan's theorem to the above expression, we can break the over-bar that covering both terms and change the sign between them as follows:

$$X = \overline{\overline{C}} \bullet \overline{D}$$

$$X = C \bullet \overline{D}$$

Example:

Simplify the following expression:

$$Y = (A + B)(A + \overline{B})$$

$$= AA + A\overline{B} + AB + B\overline{B}$$

$$= A + A(\overline{B} + B)$$

$$= A + A$$

$$= A$$

Or

$$
\begin{aligned}
Y &= AA + A\overline{B} + AB + B\overline{B} \\
&= AA + A\overline{B} + AB \\
&= A + A\overline{B} + AB \\
&= A(1 + \overline{B} + B) \\
&= A \bullet 1 \\
&= A
\end{aligned}
$$

Example:

$$
\begin{aligned}
Y &= ABC + A\overline{B}C + AB\overline{C} \\
&= AB(C + \overline{C}) + A\overline{B}C \\
&= AB + A\overline{B}C \\
&= A(B + \overline{B}C) \\
&= A(B + C)
\end{aligned}
$$

Example:

$$
\begin{aligned}
Y &= ABC + A\overline{B}\ \overline{(\overline{A}\ \overline{C})} \\
&= ABC + A\overline{B}\ (\overline{\overline{A}} + \overline{\overline{C}}) \\
&= ABC + A\overline{B}\ (A + C) \\
&= ABC + A\overline{B} + A\overline{B}C \\
&= AC(B + \overline{B}) + A\overline{B} \\
&= AC + A\overline{B} \\
&= A\ (C + \overline{B})
\end{aligned}
$$

- Determine if the following equation is valid (Derived from the truth table)

$$\overline{X1}\ \overline{X3} + X2\ X3 + X1\ \overline{X2} = \overline{X1}\ X2 + X1\ X3 + \overline{X2}\ \overline{X3}$$

Left-Hand Side (LHS)

Row number	x_1	x_2	x_3	$\bar{x}_1 \bar{x}_3$	$x_2 x_3$	$x_1 \bar{x}_2$	f
0	0	0	0	1	0	0	1
1	0	0	1	0	0	0	0
2	0	1	0	1	0	0	1
3	0	1	1	0	1	0	1
4	1	0	0	0	0	1	1
5	1	0	1	0	0	1	1
6	1	1	0	0	0	0	0
7	1	1	1	0	1	0	1

Right-Hand Side (RHS)

Row number	x_1	x_2	x_3	$\bar{x}_1 x_2$	$x_1 x_3$	$\bar{x}_2 \bar{x}_3$	f
0	0	0	0	0	0	1	1
1	0	0	1	0	0	0	0
2	0	1	0	1	0	0	1
3	0	1	1	1	0	0	1
4	1	0	0	0	0	1	1
5	1	0	1	0	1	0	1
6	1	1	0	0	0	0	0
7	1	1	1	0	1	0	1

$$\bar{x}_1\bar{x}_3 + x_2x_3 + x_1\bar{x}_2 \overset{?}{=} \bar{x}_1x_2 + x_1x_3 + \bar{x}_2\bar{x}_3$$

LHS

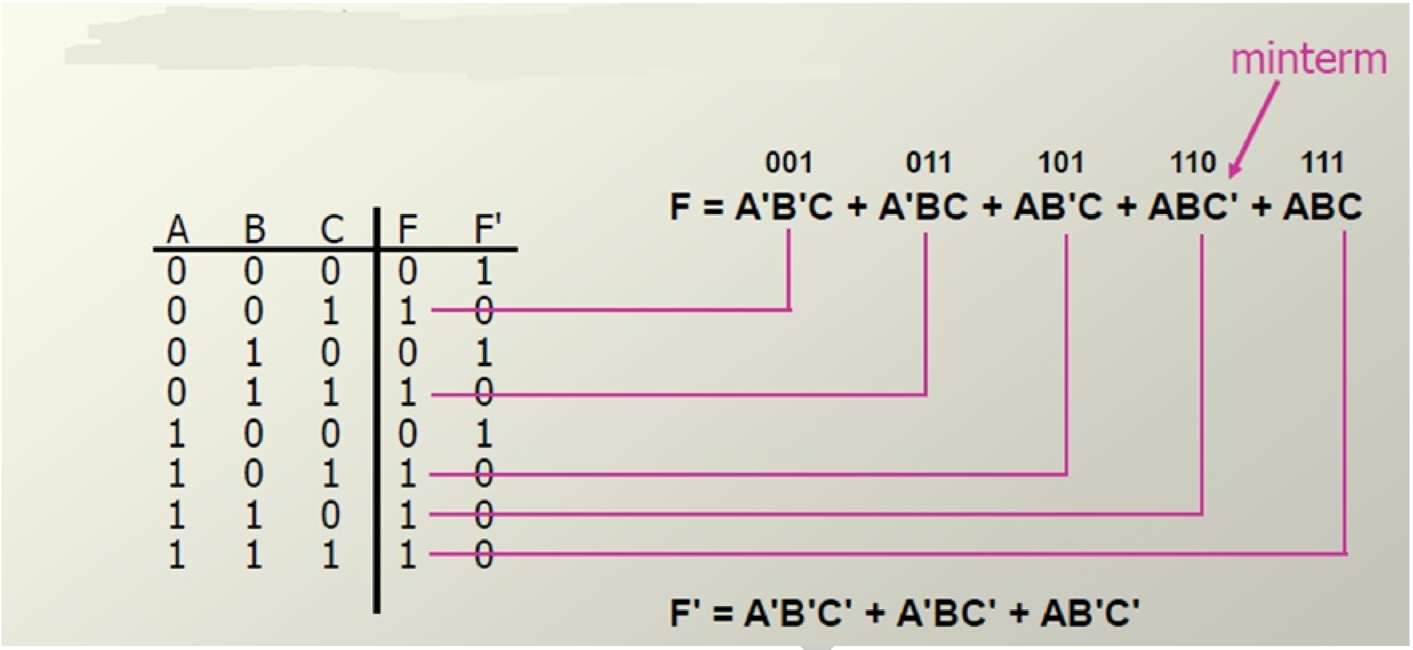
f
1
0
1
1
1
1
0
1

RHS

f
1
0
1
1
1
1
0
1

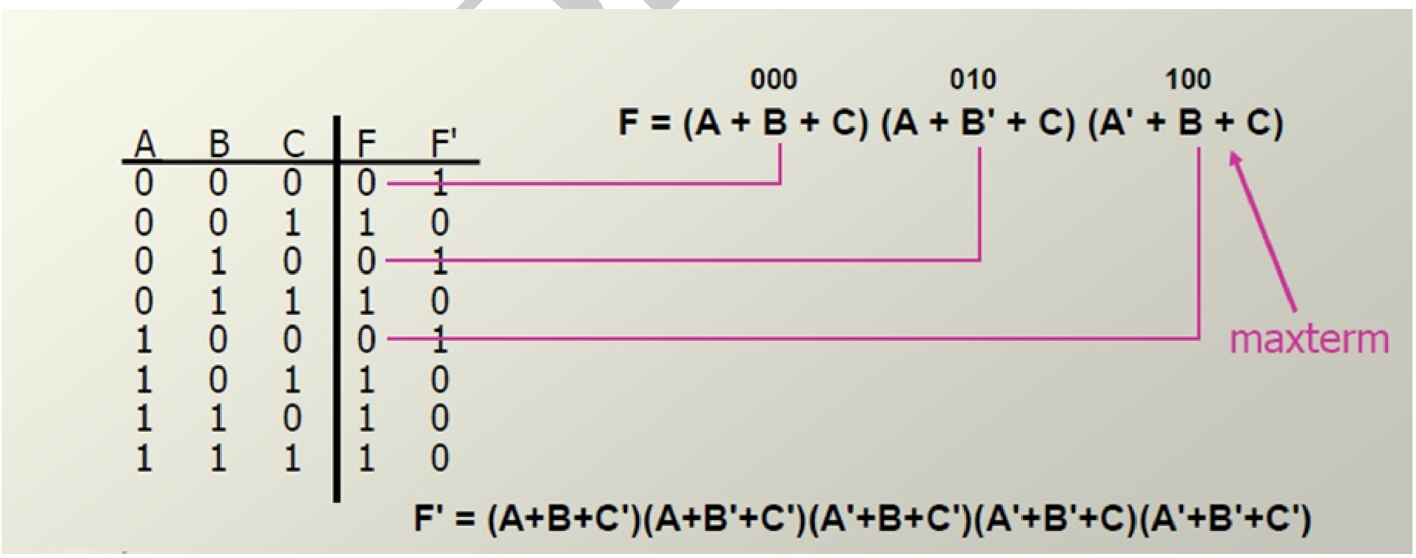
■ **Sum of Product (SOP) and Product of Sum (POS) forms**

- **Sum of Product (SOP)**



A	B	C	F	F'
0	0	0	0	1
0	0	1	1	0
0	1	0	0	1
0	1	1	1	0
1	0	0	0	1
1	0	1	1	0
1	1	0	1	0
1	1	1	1	0

- **Product of Sum (POS)**

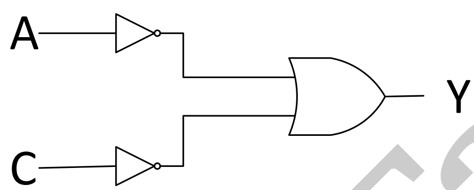


A	B	C	F	F'
0	0	0	0	1
0	0	1	1	0
0	1	0	0	1
0	1	1	1	0
1	0	0	0	1
1	0	1	1	0
1	1	0	1	0
1	1	1	1	0

Example:

Simplify the following expression, then draw the logic circuit after simplification.

$$
\begin{aligned}
Y &= \overline{A+B} + \overline{ABC} + \overline{A\overline{B}} \\
&= \overline{A}\ \overline{B} + \overline{A} + \overline{B} + \overline{C} + \overline{A} + B \\
&= \overline{A}(\overline{B} + 1 + 1) + (B + \overline{B}) + \overline{C} \\
&= \overline{A} + \overline{C}
\end{aligned}
$$



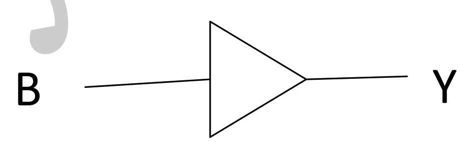
Example:

For the given truth table simplify and draw the circuit before and after simplification.

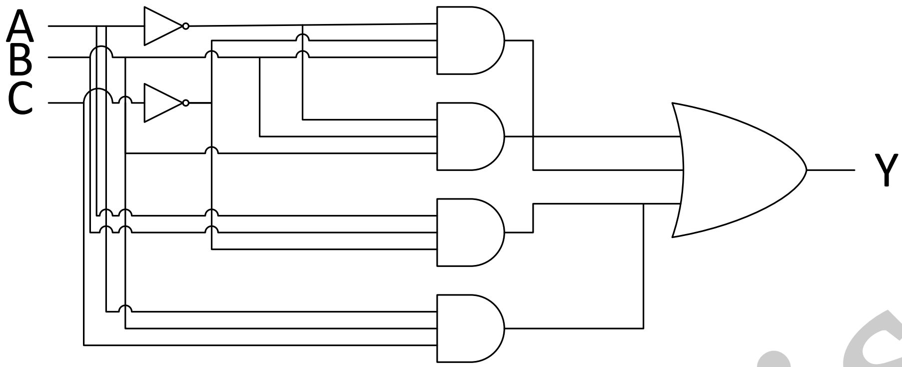
A	B	C	Y
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	1
1	1	1	1

$$
\begin{aligned}
Y &= \overline{A}B\overline{C} + \overline{A}BC + AB\overline{C} + ABC \\
&= \overline{A}B(\overline{C} + C) + AB(\overline{C} + C) \\
&= \overline{A}B + AB \\
&= B(\overline{A} + A) \\
&= B
\end{aligned}
$$

- The circuit after simplification:



- The circuit before simplification:



Exercise:

Design a logic circuit with three inputs variables (A, B, C) that will produce a HIGH ("1") output if the input is Odd number?

Exercise:

Write the equation (s) of the following logic circuit: