6- Cubic Interpolation methods:-

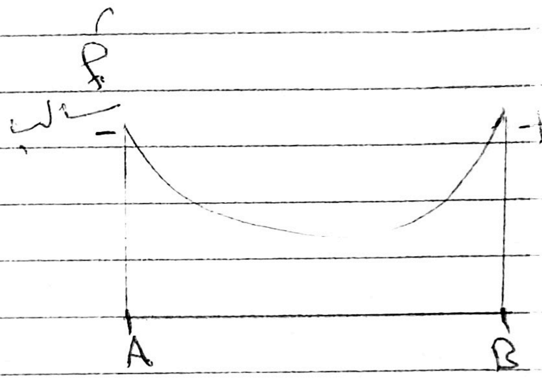
The cubic interpolation method find the minimum of one variable and multivariable.

Let:-

$$h(\lambda) = a + b\lambda + c\lambda^2 + d\lambda^3 \quad \text{———} \quad (*)$$

be the cubic function is used to approximate the function $f(\lambda)$ between points A and B, we need to find the values f_A, f'_A, f_B and f'_B in order to evaluate the constants a, b, c and d in equation (*).

$$f_A = a + bA + cA^2 + dA^3$$



$$f_B = a + bB + cB^2 + dB^3$$

$$f'_A = b + 2cA + 3dA^2$$

$$f'_B = b + 2cB + 3dB^2$$

$$a = f_A - bA - cA^2 - dA^3$$

$$b = \frac{1}{(A-B)^2}\left(B^2 f'_A + A^2 f'_B + 2ABZ\right)$$

$$c = \frac{-1}{(A-B)^2}\left[(A+B)Z + B f'_A + A f'_B\right]$$

$$d = \frac{1}{3(A-B)^2}\left(2Z + f'_A + f'_B\right)$$

$$Z = \frac{3(f_A - f_B)}{B - A} + f'_A + f'_B$$

The necessary Condition for the minimum of $h(\lambda)$ given by eq(*) is that:-

$$\frac{\partial h}{\partial \lambda} = b + 2c\lambda + 3d\lambda^2 = 0$$

$$\lambda^* = \frac{-c \pm (c^2 - 3bd)^{\frac{1}{2}}}{3d} \quad \text{(**)}$$

The sufficiency Condition for the the minimum of $h(\lambda)$ is that:-

$$\frac{\partial^2 h}{\partial \lambda^2} = 2c + 6d\lambda > 0 \quad \text{(***)}$$

by substituting b, c and d into eq(**) and (***) we obtain:-

$$\lambda^* = A + \frac{f'_A + Z \pm Q}{f'_A + f'_B + 2Z}(B - A)$$

$$Q = (Z^2 - f'_A f'_B)^{\frac{1}{2}}$$