Example:- Find the minimum of

$$f(\lambda) = \lambda^5 - 5\lambda^3 - 20\lambda + 5$$

by the cubic interpolation method. where $A = 0$, $t_0 = 0.4$ and $\epsilon = 0.05$.

Solution:-

$-$ $+$

$$f'(\lambda) = 5\lambda^4 - 15\lambda^2 - 20$$

$f'(A) = f'(0) = -20 < 0$ ✓

$f(A) = 5$

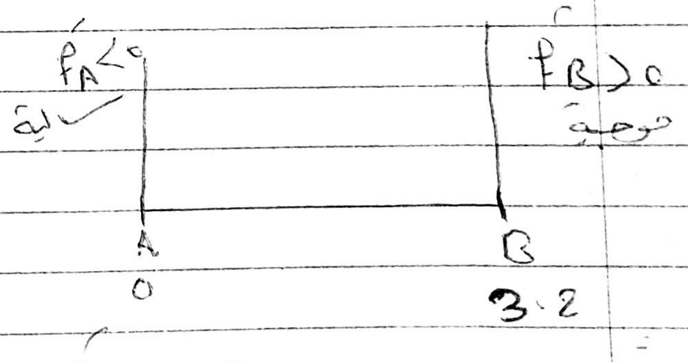
$f'(t_0) = f'(0.4) = 5(0.4)^4 - 15(0.4)^2 - 20 = -22.27$ سالب

$f'(2t_0) = f'(2(0.4)) = f'(0.8) = 5(0.8)^4 - 15(0.8)^2 - 20 = -27.55$ سالب

$f'(4t_0) = f'(4(0.4)) = f'(1.6) = 5(1.6)^4 - 15(1.6)^2 - 20 = -25.63$ سالب

$f'(8t_0) = f'(8(0.4)) = f'(3.2) = 5(3.2)^4 - 15(3.2)^2 - 20 = 350.69$ $+$

$f(3.2) = 112.7$



$A = 0$ $\quad f_A = 5$ $\quad f'_A = -20$

$B = 3.2$ $\quad f_A = 112.7$ $\quad f'_B = 350.69$

$$A < \lambda^* < B$$

iteration(1):-

$$Z = \frac{3(f_A - f_B)}{B - A} + f'_A + f'_B$$

$$= \frac{3(5 - 112.7)}{3.2 - 0} + (-20) + 350.69$$

$$= 229.72$$

$$Q = \sqrt{Z^2 - f'_A f'_B}$$

$$= \sqrt{(229.72)^2 - (-20)(350.69)}$$

$$= 244.51$$

$$\hat{\lambda} = A + \frac{f'_A + Z \pm Q}{f'_A + f'_B + 2Z}(B - A)$$

$$= 0 + \frac{(-20) + 229.72 \pm 244.5)}{-20 + 350.69 + 2(224.72)}(3.2 - 0)$$

= اما -0.14

او 1.84

نأخذ قيمة λ التي تقع ضمن الفترة $A < \hat{\lambda} < B$ أي نأخذ القيمة الموجبة 1.84 لأنها تقع ضمن الفترة

$$f'(\lambda^*) = 5(1.84)^4 - 15(1.84)^2 - 20 = -13.47$$

$$|f'(\lambda^*)| < \epsilon$$

$$|-13.47| = 13.47 > \epsilon$$