$$Z = \frac{3(f_A - f_B)}{B - A} + f'_A + f'_B$$

$$= \frac{3(1 - 0.1296)}{1.6 - 0} + (-4) + 0.864$$

$$= -1.504$$

$$Q = \sqrt{Z^2 - f'_A f'_B}$$

$$= \sqrt{(-1.504)^2 - (-4)(0.864)}$$

$$= 2.3912$$

$$\lambda^* = A + \frac{f'_A + Z \pm Q}{f'_A + f'_B + 2Z}(B - A)$$

$$= 0 + \frac{-4 + (-1.504) \pm 2.3912}{-4 + 0.864 + 2(-1.504)}(1.6 - 0)$$

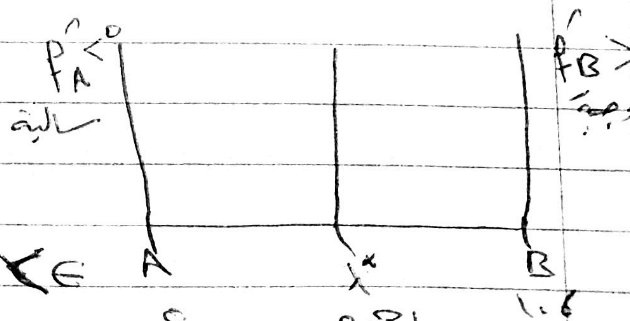
= لما 2.06

21 0.81 ✓

$$\begin{matrix} A & & \lambda^* & & B \\ 0 & < & 0.81 & < & 1.6 \end{matrix}$$

$$f'(\lambda^*) = 4(0.81 - 1)^3$$
$$= -0.0274$$

$$|f'(\lambda^*)| = |-0.0274| = 0.0274 < \epsilon$$



$\therefore$ The optimal solution $\lambda^* = 0.81$

7- Newton method :-

Consider the quadratic approximation of the function $f(\lambda)$ at $\lambda = \lambda_i$ using the taylor's series expansion :-

$$f(\lambda) = f(\lambda_i) + f'(\lambda_i)(\lambda - \lambda_i) + \frac{1}{2} f''(\lambda_i)(\lambda - \lambda_i)^2 \quad (1)$$

by setting the derivative of eq(1) equal to zero for the minimum of $f(\lambda)$, we obtain :-

$$f'(\lambda) = f'(\lambda_i) + f''(\lambda_i)(\lambda - \lambda_i) = 0 \quad (2)$$

If λ_i denotes an approximation to the minimum of $f(\lambda)$, eq(2) Can be rearranged to obtain an improved approximation as :-

$$\lambda_{i+1} = \lambda_i - \frac{f'(\lambda_i)}{f''(\lambda_i)} \quad (3)$$

The iterative process of newton method Can be described by the following steps :-

1- Given the starting point λ_0 and ϵ.

2- Evaluate $f'(\lambda)$ and $f''(\lambda)$ at λ_0

3- Find the new approximate Solution of the problem as :-

$$\lambda_{i+1} = \lambda_i - \frac{f'(\lambda_i)}{f''(\lambda_i)}$$