4- Test if $|f'(\lambda_{i+1})| \leq \epsilon$ is satisfied, $\lambda^* = \lambda_{i+1}$ and stop. otherwise set $i = i+1$ and go to step (2).

The disadvantage of newton method:-

1- The method requires both the first and second order derivatives of $f(\lambda)$.

2- If the starting point for the iterative process is not close to the true solution λ^*, the newton iterative process might diverge.

Example:- Find the minimum of the function

$$f(\lambda) = 2\lambda^2 + \frac{16}{\lambda}$$

Using newton method with the starting point $\lambda_0 = 1$. Use $\epsilon = 0.05$ for checking the convergence.

Solution:-

$$f'(\lambda) = 4\lambda - \frac{16}{\lambda^2}$$

$$f''(\lambda) = 4 + \frac{32}{\lambda^3}$$

iteration (1):-

$$f'(\lambda_0) = f'(1) = -12$$
$$f''(\lambda_0) = f''(1) = 36$$

$$\lambda_1 = \lambda_0 - \frac{f'(\lambda_0)}{f''(\lambda_0)}$$

$$= 1 - \frac{-12}{36}$$

$$= 1.33$$

$|f'(\lambda_1)| \leq \epsilon$

$f'(\lambda_1) = f'(1.33) = -3.73$

$|-3.73| = 3.73 > \epsilon$

iteration (2):-

$f'(\lambda_1) = f'(1.33) = -3.73$

$f''(\lambda_1) = f''(1.33) = 17.6$

$$\lambda_2 = \lambda_1 - \frac{f'(\lambda_1)}{f''(\lambda_1)}$$

$$= 1.33 - \frac{-3.73}{17.6} = 1.54$$

$f'(\lambda_2) = f'(1.54) = -0.59$

$|f'(1.54)| \leq \epsilon$

$|-0.59| = 0.59 > \epsilon$

iteration (3):-

$f'(\lambda_2) = f'(1.54) = -0.59$

$f''(\lambda_2) = f''(1.54) = 12.76$