$$\lambda_3 = \lambda_2 - \frac{f'(\lambda_2)}{f''(\lambda_2)}$$

$$= 1.54 - \frac{-0.59}{12.76} = 1.59$$

$f'(\lambda_3) = f'(1.59) = 0.03$

$|f'(\lambda_3)| \le \epsilon$

$|0.03| = 0.03 < \epsilon$

$\therefore$ The optimal solution $\lambda^* = 1.59$

8- Quasi newton method:-

If the function being minimized $f(\lambda)$ is not available in closed form or is difficult to differentiate, the derivatives $f'(\lambda)$ and $f''(\lambda)$ in equation:-

$$\lambda_{i+1} = \lambda_i - \frac{f'(\lambda_i)}{f''(\lambda_i)} \quad \text{—} (1)$$

can be approximated by the finite difference formulas as:-

$$f'(\lambda_i) = \frac{f_i^+ - f_i^-}{2\Delta\lambda} \quad \text{—} (2)$$

$$f''(\lambda_i) = \frac{f_i^+ - 2f_i + f_i^-}{\Delta\lambda^2} \quad \text{—} (3)$$

where $\Delta\lambda$ is a small step size. Substitution of eq(2) and eq(3) into eq(1) leads to:-

$$\lambda_{i+1} = \lambda_i - \frac{\Delta\lambda(f_i^+ - f_i^-)}{2[f_i^+ - 2f_i + f_i^-]} \quad (4)$$

where $f_i = f(\lambda_i)$, $f_i^+ = f(\lambda_i + \Delta\lambda)$ and

$f_i^- = f(\lambda_i - \Delta\lambda)$

The iterative process indicated by eq(4) is known as the quasi newton method.

The basic steps of Quasi newton method are:-

1- Given λ_0 , $\Delta\lambda$ and ϵ

2- Evaluate f_0, f_0^+ and f_0^-

where $f_0 = f(\lambda_0)$, $f_0^+ = f(\lambda_0 + \Delta\lambda)$ and

$f_0^- = f(\lambda_0 - \Delta\lambda)$

3- Find the new point:-

$$\lambda_{i+1} = \lambda_i - \frac{\Delta\lambda(f_i^+ - f_i^-)}{2(f_i^+ - 2f_i + f_i^-)}$$

4- Test if $\left|\frac{f_{i+1}^+ - f_{i+1}^-}{2\Delta\lambda}\right| \leq \epsilon$, stop. otherwise go to step(2).