Example:- Find the minimum of the function

$$f(\lambda) = 2\lambda^2 + \frac{16}{\lambda}$$

with the starting point $\lambda_0 = 1$ and the step size $\Delta\lambda = 0.01$. Use $\epsilon = 0.05$ for checking the convergence.

Solution:- $\lambda_0 = 1$ $\Delta\lambda = 0.01$ $\epsilon = 0.05$

iteration (1):-

$$f_0 = f(\lambda_0) = f(1) = 18$$

$$f_0^+ = f(\lambda_0 + \Delta\lambda) = f(1 + 0.01)$$
$$= f(1.01) = 2(1.01)^2 + \frac{16}{1.01}$$
$$= 17.8818$$

$$f_0^- = f(\lambda_0 - \Delta\lambda) = f(1 - 0.01)$$
$$= f(0.99) = 2(0.99)^2 + \frac{16}{0.99}$$
$$= 18.1218$$

$$\lambda_1 = \lambda_0 - \frac{\Delta\lambda\,(f_0^+ - f_0^-)}{2(f_0^+ - 2f_0 + f_0^-)}$$

$$\lambda_1 = 1 - \frac{0.01\,(17.8818 - 18.1218)}{2\,[17.8818 - 2(18) + 18.1218]}$$

$$\lambda_1 = 1.33$$

$$\left|\frac{f_1^+ - f_1^-}{2\Delta\lambda}\right| \le \epsilon$$

$f_1^+ = f(\lambda_1 + \Delta\lambda) = f(1.33 + 0.01)$

$= f(1.34) = 2(1.34)^2 + \frac{16}{1.34}$

$= 15.5315$

$f_1^- = f(\lambda_1 - \Delta\lambda) = f(1.33 - 0.01)$

$= f(1.32) = 2(1.32)^2 + \frac{16}{1.32}$

$= 15.6060$

$\left| \frac{f_1^+ - f_1^-}{2\Delta\lambda} \right| = \left| \frac{15.5315 - 15.6060}{2(0.01)} \right| = 3.725 > \epsilon$

iteration (2):-

$\lambda_1 = 1.33$, $f_1^+ = 15.5315$, $f_1^- = 15.6060$

$f_1 = f(\lambda_1) = f(1.33) = 2(1.33) + \frac{16}{1.33}$

$= 15.5679$

$$\lambda_2 = \lambda_1 - \frac{\Delta\lambda\,(f_1^+ - f_1^-)}{2\,[f_1^+ - 2f_1 + f_1^-]}$$

$$\lambda_2 = 1.33 - \frac{0.01\,(15.5315 - 15.6060)}{2\,[15.5315 - 2(15.5679) + 15.6060]}$$

$\lambda_2 = 1.55$

$f_2^+ = f(\lambda_2 + \Delta\lambda) = f(1.55 + 0.01)$

$= f(1.56) = 2(1.56) + \frac{16}{1.56}$

$= 15.1236$