Lecture (4)

- Harmonic mean
- Mean deviation
- Some examples

EX(2):-

Given the following prob. dist^{n} table:-

X	0	1	2	3
$p(X)$	$\frac{1}{4}$	0	$\frac{1}{4}$	$\frac{1}{2}$

Find: μ, μ_1', μ_2', μ_1, μ_2, σ^2, σ.

sol:-

$$\mu = E(X) = \sum_{x=0}^{3} x\, p(X=x) = 0\left(\frac{1}{4}\right) + (1)(0) + 2\left(\frac{1}{4}\right) + 3\left(\frac{1}{2}\right) = 2$$

$$\mu_1' = E(X) = \mu = 2.$$

$$\mu_2' = EX^2 = \sum_{x=0}^{3} x^2 p(x) = 0^2 p(X=0) + 1^2 p(X=1) + 2^2 p(X=2)$$

$$+ 3^2 p(X=3) = 4\left(\frac{1}{4}\right) + 9\left(\frac{1}{2}\right) = 1 + \frac{9}{2} = 5.5$$

$$\mu_1 = E(X-\mu) = E(X) - \mu = 2 - 2 = 0$$

$$\mu_2 = E(X-\mu)^2 = EX^2 - \mu^2 = 5.5 - 4 = 1.5$$

$$\sigma^2 = E(X-\mu)^2 = 1.5, \quad \sigma = \sqrt{1.5} = 1.223$$

4) The harmonic mean (H):-

It is a special mathematical expectation which is obtained by taking $\left(E\left(\frac{1}{X}\right)\right)^{-1} = \left(E(U(x))\right)^{-1}$ ie

$$H = \left(E\left(\frac{1}{X}\right)\right)^{-1} \quad \text{or} \quad \frac{1}{H} = E\left(\frac{1}{X}\right)$$

This means

$$\frac{1}{H} = \begin{cases} \sum_{x} \frac{1}{x} f(x) & \text{if } X \text{ is a discrete r.v.} \\ \int_{x} \frac{1}{x} f(x)\,dx & \text{if } X \text{ is a continious r.v.} \end{cases}$$

ex(1):-

If a r.v. X has the following p.m.f

$$f(x) = \begin{cases} \frac{1}{8} & x = 1, 2, \ldots, 8 \\ 0 & o.w. \end{cases}$$

Find the harmonic mean for the distribution

Sol:-

$$\frac{1}{H} = \sum_{x=1}^{8} \frac{1}{x} f(x) = \frac{1}{8} \sum_{x=1}^{8} \frac{1}{x} = \frac{1}{8}\left[1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{8}\right] = 0.339 \Rightarrow H = 2.943.$$

ex:- A r.v. X has the following p.d.f

$$f(x) = \begin{cases} \frac{1}{x^2} & 1 < x < \infty \\ 0 & o.w. \end{cases}$$

Find the harmonic mean for the distⁿ.

sol:-

$$E\left(\frac{1}{X}\right) = \int_1^\infty \frac{1}{x^3}\, dx = -\frac{1}{2x^2}\Big|_1^\infty = \frac{1}{2}$$

$\therefore H = 2$

5) The mean deviation (Md) الانحراف المطلق

The mean deviation is defined as the first absoluate central moment ie:

$$Md = E|X - E(X)| \qquad , Md \geq 0$$

ex(1):-

If a r.v. X has the following p.m.f

$$f(x) = \begin{cases} \frac{1}{7} & x = 1, 2, 3, \dots, 7 \\ 0 & o.w. \end{cases}$$

Find the mean deviation of the distⁿ.

sol:-

$$M_d = E|X - EX|$$

$$EX = \sum_{x=1}^{7} \frac{x}{7} = \frac{1}{7}(1+2+\dots+7) = \frac{7(7+1)}{7(2)} = 4$$

$$\therefore M_d = E|X-4| = \sum_{x=1}^{7} \frac{|x-4|}{7} = \frac{1}{7}(3+2+1+0+1+2+7)$$

$$= \frac{12}{7}$$

ex(2):-

If the p.d.f of a r.v. x has the following form:-

$$f(x) = \begin{cases} \frac{1}{10} & -3 < x < 7 \\ 0 & o.w. \end{cases}$$

Find the mean deviation of X and the mean deviation of $Y = 4 - 3X$

Sol:-

$$E(X) = \int_{-3}^{7} \frac{x}{10}\, dx = \frac{x^2}{20}\Big|_{-3}^{7} = 2$$

$$\therefore Md = \int_{-3}^{7} |X-2| \cdot \frac{1}{10}\, dx = \frac{1}{10}\int_{-3}^{7} |X-2|\, dx$$

$$= \frac{1}{10}\left[\int_{-3}^{2} -(X-2)\,dx + \int_{2}^{7} (X-2)\,dx\right]$$

$$= \frac{1}{10}\left[-\frac{X^2}{2}\Big|_{-3}^{2} + 2X\Big|_{-3}^{2} + \frac{X^2}{2}\Big|_{2}^{7} - 2X\Big|_{2}^{7}\right]$$

$$= \frac{1}{10}\left[-\frac{(4-9)}{2} + 2(2+3) + \frac{49-4}{2} - 2(7-2)\right] = 2.5$$

Before finding $M_d(Y)$, we have the following property:-

If a and b are two constants and if we define a r.v. $Y = aX \mp b$ then the mean deviation of Y is

$$M_d(Y) = |a|\, M_d(X)$$

$$\therefore M_d(4-3X) = |-3|\, M_d(X) = 3(2.5) = 7.5$$

EX (1) H.W.

If a r.v. X follow the folowing p.d.f

$$f(x) = \begin{cases} \dfrac{x-c}{18} & -2<x<4 \\ 0 & o.w \end{cases}$$

Find ; The constant c, EX, $E(X-4)^2$, $E(X-EX)^3$ and V(X).

Sol:-

$$1 = \int_{-2}^{4} f(x)\,dx \Rightarrow 1 = \frac{1}{18}\int_{-2}^{4} (x-c)\,dx$$

$$\Rightarrow 18 = \frac{x^2}{2}\Big|_{-2}^{4} + cx\Big|_{-2}^{4} \Rightarrow c = 2$$

$$\therefore f(x) = \begin{cases} \dfrac{x-2}{18} & -2<x<4 \\ 0 & o.w. \end{cases}$$

$$EX = \int_{-2}^{4} \frac{x-2}{18}\,dx = \frac{1}{18}\left[\frac{x^2}{2}\Big|_{-2}^{4} - 2x\Big|_{-2}^{4}\right] = \frac{1}{18}(6-12)$$

$$= -\frac{1}{3}$$

$$E(X-4)^2 = EX^2 - 8EX + 16$$

$$EX^2 = \int_{-2}^{4} x^2 f(x)\,dx = \frac{1}{18}\int_{-2}^{4}(x^3-2x^2)\,dx$$

$$= \frac{1}{18}\left[\frac{x^4}{4}\Big|_{-2}^{4} - 2\,\frac{x^3}{3}\Big|_{-2}^{4}\right] = \frac{12}{18} = \frac{2}{3}$$

$$\therefore E(X-4)^2 = \frac{2}{3} + 8\left(\frac{1}{3}\right) + 16 = 19.333$$

$$E(X-EX)^3 = E(X+\frac{1}{3})^2 = E(X+\frac{1}{3})(X^2+\frac{2}{3}X+\frac{1}{9})$$

$$= E(X^3+\frac{2}{3}X^2+\frac{1}{9}X+\frac{1}{3}X^2+\frac{2}{9}X+\frac{1}{27})$$

$$= E(X^3+X^2+\frac{1}{3}X+\frac{1}{27})$$

$$EX^3 = \int_{-2}^{4} x^3 \cdot \frac{x-2}{18}\,dx = \frac{1}{18}\left[\frac{x^5}{5}\Big|_{-2}^{4} - \frac{x^4}{2}\Big|_{-2}^{4}\right.$$

$$= \frac{1}{18}(211.2-120) = 5.067$$

$$V(X) = EX^2-(EX)^2 = \frac{2}{3}-(-\frac{1}{3})^2 = \frac{5}{9}$$

EX(2):- H.W.

Let $f(x)=2x$, $0<x<1$ and zero otherwise, be the p.d.f of X

a- Compute $E(\frac{1}{X})$

b- Find the c.d.f and the p.d.f of $Y=\frac{1}{X}$

c- Comput $E(Y)$ and compare this result with the answer obtained in (a).

Sol:-

a- $E(\frac{1}{X}) = 2\int_0^1 dx = 2$

b- $F(Y) = P(Y\leq y) = P(\frac{1}{X}\leq y) = P(X\geq\frac{1}{y})$

$$= 1-P(X\geq\frac{1}{y}) = 1-\int_0^{\frac{1}{y}} 2x\,dx$$

$$= 1-2(\frac{x^2}{2})\Big|_0^{\frac{1}{y}} = 1-\frac{1}{y^2} \qquad 1<y<\infty$$