(11) اخي طرة

Ex: show that $|\sin b - \sin a| \le |b-a|$ p:152

$\forall\ a, b \in \mathbb{R}$

solution:

Let $f(x) = \sin x$, $x \in [a,b]$, $f'(x) = \cos x$

1) $f(x)$ is continuous on $[a,b]$

2) $f'(x)$ exist on (a,b)

$\Rightarrow f$ satisfy M.V.T on $[a,b]$

$\Rightarrow \exists\ c \in (a,b)$ s.t $f'(c) = \dfrac{f(b)-f(a)}{b-a}$

$$\Rightarrow \frac{\sin b - \sin a}{b-a} = \cos c$$

by taking |المقياس| for the both sides

$$\Rightarrow |\cos c| = \left| \frac{\sin b - \sin a}{b-a} \right|$$

but $|\cos c| \le 1$ since $-1 \le \cos x \le +1$

$$\left| \frac{\sin b - \sin a}{b-a} \right| \le 1$$

$$\Rightarrow |\sin b - \sin a| \le |b-a|$$

Increasing Functions and Decreasing Function

Def: let $f(x)$ be a function define in the interval I

1) then f is increasing function in I iff, $\forall$ paire of of numbers x_1 and x_2 in I

$$x_1 < x_2 \Rightarrow f(x_1) < f(x_2)$$

2) f is decreasing function in I iff $\forall$ pair of number x_1 and x_2 in I

$$x_1 < x_2 \Rightarrow f(x_1) > f(x_2)$$

Theorem: If $f(x)$ is continuous function in I and differentiable $\forall$ point in I, then

1) If $f'(x) > 0$ $\forall$ x in I then f increasing in I

2 If $f'(x) < 0$ $\forall$ x in I, then f decreasing in I

Example: $f(x) = 3x^2 - 8x^3 - 6x^2 + 24x - 10$
Is the function is increasing or decreasing

$f(x) = 3x^4 - 8x^3 - 6x^2 + 24x - 10$

solution — GP ص٤٩ الكتاب

$f'(x) = 12x^3 - 24x^2 - 12x + 24$

$= 12(x^3 - 2x^2 - x + 2)$

$$
\begin{array}{r}
x^2 - x - 2 \\
(x-1)\overline{)\,x^3 - 2x^2 - x + 2} \\
-x^3 + x^2 \\
\hline
-x^2 - x + 2 \\
+x^2 - x \\
\hline
-2x + 2 \\
+2x - 2 \\
\hline
0 \quad 0
\end{array}
$$

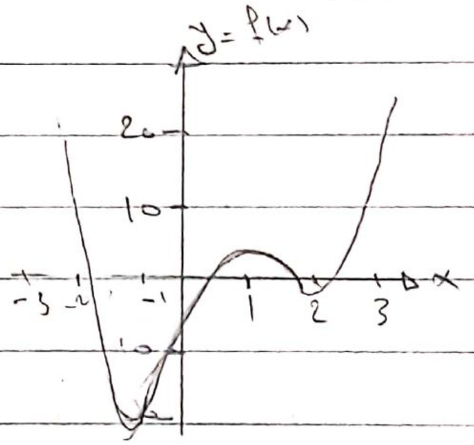
$f'(x) = 12(x-1)(x^2 - x - 2)$

$= 12(x-1)(x-2)(x+1)$

$= 12(x+1)(x-1)(x-2)$

1) $(x+1) < 0 \Rightarrow x < -1$

$(x+1) > 0 \Rightarrow x > -1$



2) $(x-1) < 0 \Rightarrow x < 1$

$(x-1) > 0 \Rightarrow x > 1$

3) $(x-2) < 0 \Rightarrow x < 2$

$(x-2) > 0 \Rightarrow x > 2$

لذا فان $f'(x) < 0$ اذا كانت (متناقصة) اذا كانت

1) $x < -1, \quad x < 1, \quad x < 2$

$f'(x) > 0$ اذا كانت

1) $x > -1$

$x > 1$

$x > 2$

linear Approximation and Differentials 191
Thomas

التقريبات الخطية والمشتقات

Def.: If $y = f(x)$ is differentiable at $x = a$, then

$$L(x) = f(a) + f'(a) \cdot (x - a)$$

is the linearization of f at a, the approximation

$$f(x) \approx L(x)$$

is the standard linear approximation of f at a

EX: Find the linearization of $f(x) = \sqrt{1+x}$ at $x = 0$

اولاً نحسب مشتقة الدالة عند $a=0$

$$f(x) = \sqrt{1+x}$$

$$f'(x) = \frac{1}{2\sqrt{1+x}}$$

نعوض $x=0$ في المشتقة

$$f'(0) = \frac{1}{2}$$

$$L(x) = f(a) + f'(a)(x - a)$$

نطبق القانون

$$L(x) = 1 + \frac{1}{2}(x)$$

$$\therefore L(x) \approx f(x)$$

$$\sqrt{1+x} \approx 1 + \frac{x}{2}$$

لكي حساب هذه الدالة التقريبية نطبق لبعض القيم العددية

$\sqrt{1.2} = 1 + \frac{0.2}{2} = 1.10$

$\sqrt{1.05} = 1 + \frac{0.05}{2} = 1.025$

$\sqrt{1.005} = 1 + \frac{0.005}{2} = 1.0025$

Ex: Find the linearization of $f(x) = \sqrt{1+x}$ at $x = 3$

$f(3) = \sqrt{1+3} = \sqrt{4} = 2$

$f'(x) = \frac{1}{2\sqrt{1+x}}$, $f'(3) = \frac{1}{4}$

$$L(x) = f(a) + f'(a)(x-a)$$
$$= 2 + \frac{x-3}{4} = 2 + \frac{x}{4} - \frac{3}{4}$$
$$= \frac{5}{4} + \frac{x}{4}$$

Ex: Find the Linearization of $f(x) = \tan x$ at $x = 0$

نستخدم المعادلة الآتية

$$L(x) = f(a) + f'(a).(x-a)$$

$f(0) = \tan(0) = 0$, $f'(x) = \sec^2(x) = \frac{1}{\cos^2 x} =$

$f'(0) = \frac{1}{\cos^2(0)} = 1$

$L(x) = 1(x) - 0) = x$

Linearization at $x=0$

function x	linearization $L(x)$
$\sin x$	x
$\cos x$	1
$\tan x$	x

Ex: find the linearization of $f(x)=\cos x$ at $x=\frac{\pi}{2}$

Sollution:- نستخدم المعادلة الآتية

$$L(x)=f(a)+f'(a)(x-a)$$

$f(x)=\cos x \quad , \quad f(\pi/2)=\cos\frac{\pi}{2}=0$

$f'(x)=-\sin x \quad , \quad f'(\pi/2)=-\sin(\frac{\pi}{2})=-1$

$\therefore \; L(x)=0-1*(x-\pi/2)$

$\therefore \; L(x)=\frac{\pi}{2}-x$

$\cos x=\frac{\pi}{2}-x$