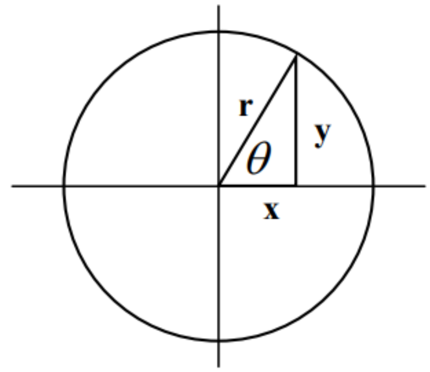
TRIGONOMETRIC FUNCTIONS الدوال المثلثية



$$\sin\theta = \frac{\text{Opposite}}{\text{Hypotenuse}} = \frac{y}{r}$$

$$\cos\theta = \frac{Adjance}{\text{Hypotenuse}} = \frac{x}{r}$$

By phythaghore theorem we get :

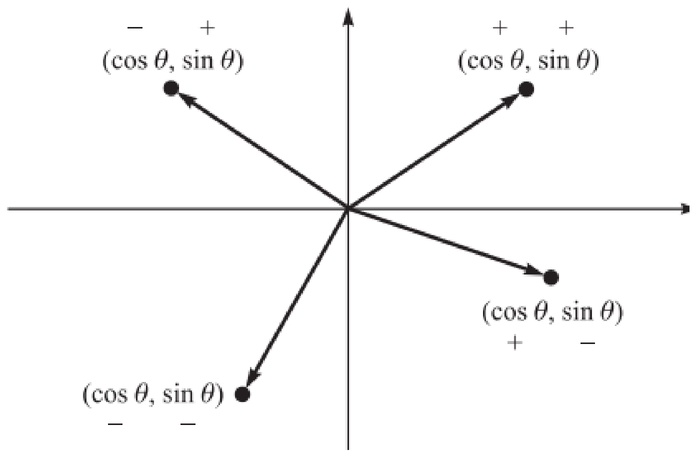
$$\sin\theta = \frac{y}{r} \Rightarrow \sin^2\theta = \frac{y^2}{r^2} \quad , \quad \cos\theta = \frac{x}{r} \Rightarrow \cos^2\theta = \frac{x^2}{r^2}$$

$$\sin^2\theta + \cos^2\theta = \frac{y^2}{r^2} + \frac{x^2}{r^2} = \frac{x^2 + y^2}{r^2} = \frac{r^2}{r^2} = 1$$

$$\therefore \quad \boxed{\sin^2\theta + \cos^2\theta = 1}$$

Trigonometric functions are also known as Circular Functions(الدوال الدائرية) can be simply defined as the functions of an angle of a triangle. It means that the relationship between the angles and sides of a triangle are given by these trig functions. The basic trigonometric functions are sine, cosine, tangent, cotangent, secant and cosecant. Also, read trigonometric identities here.

تُعرف الدوال المثلثية أيضًا باسم الدوال الدائرية ويمكن تعريفها ببساطة على أنها دوال زاوية المثلث. وهذا يعني أن العلاقة بين زوايا المثلث وأضلاعه تعطى من خلال هذه الدوال المثلثية



	0	45	30	60	90	π	$\frac{3\pi}{2}$	2π
Sin	**0**	$1/\sqrt{2}$	$1/2$	$\sqrt{3}/2$	**1**	**0**	**-1**	**0**
Cos	**1**	$1/\sqrt{2}$	$\sqrt{3}/2$	$1/2$	**0**	**-1**	**0**	**1**
Tan	**0**	**1**	$1/\sqrt{3}$	$\sqrt{3}$	∞	**0**	∞	**0**

Some Rules of Trig. Fun.

$$\sin^2\theta + \cos^2\theta = 1$$

$$\sec^2\theta = 1 + \tan^2\theta$$

$$c\,\mathrm{sc}^2\theta = 1 + co\,\mathrm{t}^2\theta$$

$$\sin(A \pm B) = \sin A\cos B \pm \cos A\sin B$$

$$\cos(A \pm B) = \cos A\cos B \mp \sin A\sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\tan(2A) = \frac{2\tan A}{1 - \tan^2 A}$$

$$\sin 2A = 2\sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A$$

$$\cos 2A = 1 - 2\sin^2 A$$

$$\cos 2A = 2\cos^2 A - 1$$

$$\sin^2 A = \frac{1}{2}(1 - \cos 2A)$$

$$\cos^2 A = \frac{1}{2}(1 + \cos 2A)$$

$$\sin A - \sin B = 2\sin\frac{A - B}{2}\cos\frac{A + B}{2}$$

$$\sin(A + B) - \sin(A - B) = 2\cos A \sin B$$

<u>EXAM :</u>

$$\sin(120) = \sin(\pi - 60) = +\sin 60 = \frac{\sqrt{3}}{2}$$

$$\sin(240) = \sin(\pi + 60) = -\sin 60 = -\frac{\sqrt{3}}{2}$$

$$\sin(300) = \sin(2\pi - 60) = -\sin 60 = -\frac{\sqrt{3}}{2}$$

$$\sin(330) = \sin(2\pi - 30) = -\sin 30 = -\frac{1}{2}$$

$$\sin(\frac{\pi}{2} - \theta) = \sin\frac{\pi}{2}\cos\theta - \cos\frac{\pi}{2}\sin\theta = \cos\theta$$

$$\cos(\frac{\pi}{2} - \theta) = \cos\frac{\pi}{2}\cos\theta + \sin\frac{\pi}{2}\sin\theta = \sin\theta$$

$\boxed{1}$ $\frac{d}{dx}\sin x = \cos x \Rightarrow in\ Gen. \Rightarrow \frac{d}{dx}\sin u = \cos u\ \frac{du}{dx}$

$\boxed{2}$ $\frac{d}{dx}\cos x = -\sin x \Rightarrow in\ Gen. \Rightarrow \frac{d}{dx}\cos u = -\sin u\ \frac{du}{dx}$

$\boxed{3}$ $\frac{d}{dx}\tan x = \sec^2 x \Rightarrow in\ Gen. \Rightarrow \frac{d}{dx}\tan u = \sec^2 u\ \frac{du}{dx}$

$\boxed{4}$ $\frac{d}{dx}\cot x = -\csc^2 x \Rightarrow in\ Gen. \Rightarrow \frac{d}{dx}\cot u = -\csc^2 u\ \frac{du}{dx}$

$\boxed{5}$ $\frac{d}{dx}\sec x = \sec x\ \tan x \Rightarrow in\ Gen. \Rightarrow \frac{d}{dx}\sec u = \sec u\ \tan u\ \frac{du}{dx}$

$\boxed{6}$ $\frac{d}{dx}\csc x = -\csc x\ \cot x \Rightarrow in\ Gen. \Rightarrow \frac{d}{dx}\csc u = -\csc u\ \cot u\ \frac{du}{dx}$

Proof :

$$\boxed{1}\ \frac{d}{dx}\sin x = \lim_{\Delta x \to 0}\frac{\sin(x+\Delta x)-\sin x}{\Delta x}$$

$$= \lim_{\Delta x \to 0}\frac{\sin x\ \cos\Delta x + \sin\Delta x\ \cos x - \sin x}{\Delta x}$$

$$= \lim_{\Delta x \to 0}\frac{-\sin x\,[1-\cos\Delta x] + \sin\Delta x\ \cos x}{\Delta x}$$

$$= \lim_{\Delta x \to 0}\frac{-\sin x\,[1-\cos\Delta x]}{\Delta x} + \lim_{\Delta x \to 0}\frac{\sin\Delta x\ \cos x}{\Delta x}$$

$$= -\sin x\ \lim_{\Delta x \to 0}\frac{[1-\cos\Delta x]}{\Delta x} + \cos x\ \lim_{\Delta x \to 0}\frac{\sin\Delta x}{\Delta x}$$

$$= -\sin x\,(0) + \cos x\,(1) = \cos x$$

$$\boxed{2}\ \frac{d}{dx}\cos x = \lim_{\Delta x \to 0} \frac{\cos(x + \Delta x) - \cos x}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\cos x \cos \Delta x - \sin x \sin \Delta x - \cos x}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{-\cos x [1 - \cos \Delta x] - \sin x \sin \Delta x}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{-\cos x [1 - \cos \Delta x]}{\Delta x} - \lim_{\Delta x \to 0} \frac{\sin \Delta x \sin x}{\Delta x}$$

$$= -\cos x \lim_{\Delta x \to 0} \frac{[1 - \cos \Delta x]}{\Delta x} - \sin x \lim_{\Delta x \to 0} \frac{\sin \Delta x}{\Delta x}$$

$$= -\cos x\,(0) + -\sin x\,(1) = -\sin x$$

EXAM :

$$y = \cos 4x \Rightarrow \frac{dy}{dx} = -\sin 4x\,(4) = -4\sin 4x$$

$$y = e^{\tan x} \Rightarrow \frac{dy}{dx} = e^{\tan x}(\sec^2 x)$$

$$y = Ln \sin 5x \Rightarrow \frac{dy}{dx} = \frac{5\cos 5x}{\sin 5x} = 5\cot 5x$$

$$y = (x^2+1)^{\sin x}$$

$$Lny = Ln(x^2+1)^{\sin x} = \sin x \; Ln(x^2+1)$$

$$\frac{1}{y}\frac{dy}{dx} = \sin x \; \frac{2x}{x^2+1} + Ln(x^2+1).\cos x$$

$$\frac{dy}{dx} = y\left[\sin x \; \frac{2x}{x^2+1} + Ln(x^2+1).\cos x\right]$$

$$\frac{dy}{dx} = (x^2+1)^{\sin x}\left[\sin x \; \frac{2x}{x^2+1} + Ln(x^2+1).\cos x\right]$$

$$y = \frac{\sin x}{1+\cos x} \Rightarrow \frac{dy}{dx} = \frac{(1+\cos x)\cos x - \sin x(-\sin x)}{(1+\cos x)^2}$$

$$\frac{dy}{dx} = \frac{\cos x + \cos^2 x + \sin^2 x}{(1+\cos x)^2} = \frac{1+\cos x}{(1+\cos x)^2} = \frac{1}{1+\cos x}$$

EXAM :

$$\text{if } y = \sin^2 3x \text{ , prove that } \frac{dy}{dx} = 3\sin 6x$$

$$\frac{dy}{dx} = 2\sin 3x \cos 3x \,(3)$$

$$\because \quad 2\sin 3x \cos 3x = \sin 6x$$

$$\therefore \frac{dy}{dx} = \sin 6x \cdot (3) = 3\sin 6x$$

EXAM : Show that y=cosx & y=sinx are solution of the eq. $y'' + y = 0$

1) $y = \cos x \Rightarrow y' = -\sin x \Rightarrow y'' = -\cos x$

$y'' + y = -\cos x + \cos x = 0 = L.H.S$

2) $y = \sin x \Rightarrow y' = \cos x \Rightarrow y'' = -\sin x$

$y'' + y = -\sin x + \sin x = 0 = L.H.S$

Homework

(1) $\text{if } y = 2\cos^2 x - 2\cos^4 x - \frac{1}{8}$, show that $\frac{dy}{dx} = \sin 4x$

(2) show that $y = x \sin x$ is solution to

$$y'' + y = 2\cos x \quad \& \quad y^{(4)} + y'' = -2\cos x$$

(3) Find $\frac{dy}{dx}$ to

$$y = \frac{\csc x}{\tan x} \text{ , } y = \frac{1}{\cot x} \text{ , } y = \frac{\sin x \sec x}{1 + x \tan x} \text{ , } y = \frac{\cot x}{1 + \csc x}$$