Even and Odd functions

The cos and sec functions are even functions; the rest other functions are odd functions.

sin(-x) = -sin x

cos(-x) = cos x

tan(-x) = – tan x

cot(-x) = -cot x

csc(-x) = -csc x

sec(-x) = sec x

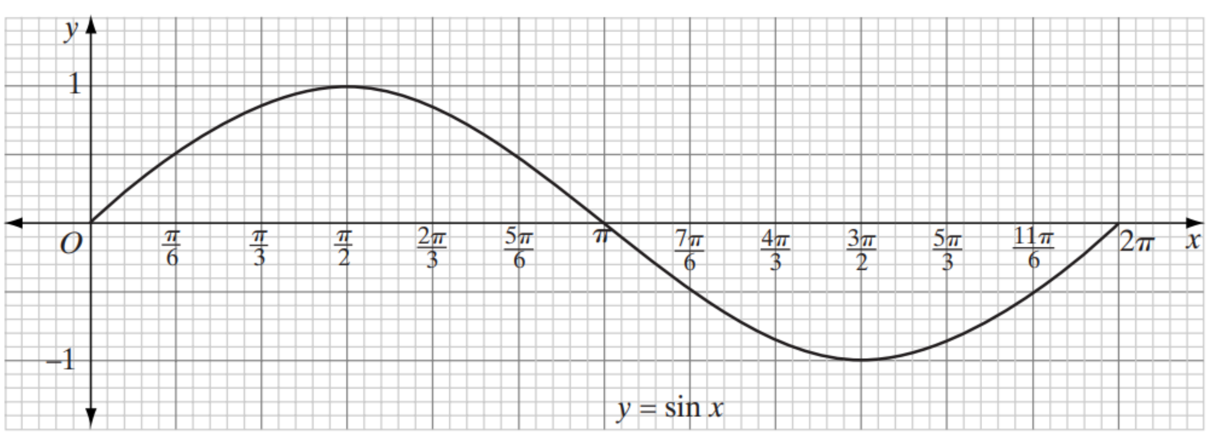
Gragh of The Sine Function

The sine function is a set of ordered pairs of real numbers. Each ordered pair can be represented as a point of the coordinate plane. The domain of the sine function is the set of real numbers, that is, every real number is a first element of one pair of the function.

To sketch the graph of the sine function, we will plot a portion of the graph using the subset of the real numbers in the interval $0 \leq x \leq 2\pi$.

$$\sin \frac{\pi}{6} = \frac{1}{2} = 0.5$$

x	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{5\pi}{6}$	π	$\frac{7\pi}{6}$	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	$\frac{11\pi}{6}$	2π
sin x	0	0.5	0.87	1	0.87	0.5	0	−0.5	−0.87	−1	−0.87	−0.5	0



The function $y = \sin x$ is called a **periodic function** with a **period** of 2π because for every x in the domain of the sine function, $\sin x = \sin (x + 2\pi)$.

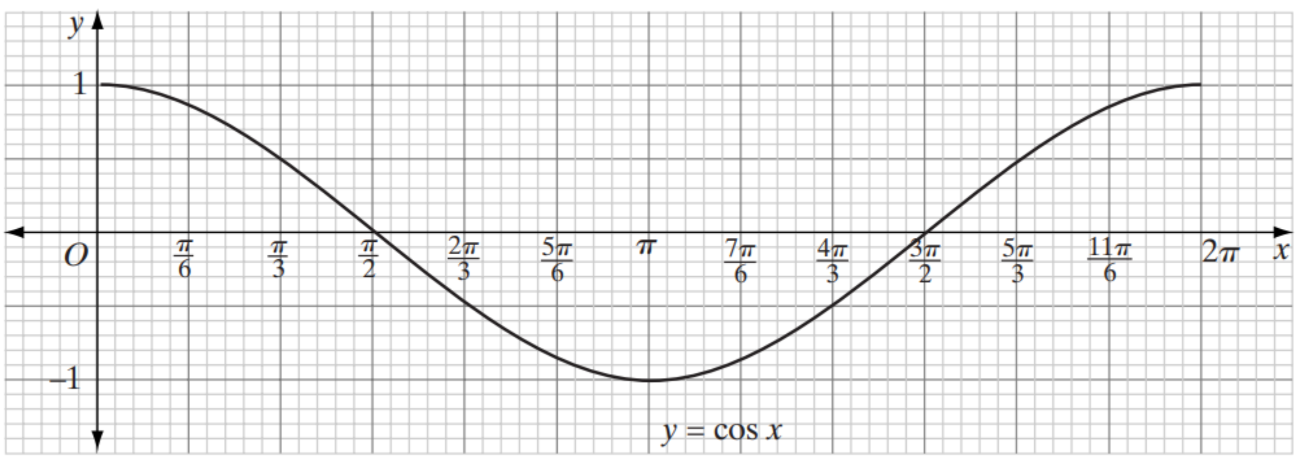
Gragh of The Cosine Function

The cosine function, like the sine function, is a set of ordered pairs of real numbers. Each ordered pair can be represented as a point of the coordinate plane. The domain of the cosine function is the set of real numbers, that is, every real number is a first element of one pair of the function.

To sketch the graph of the cosine function, we plot a portion of the graph using a subset of the real numbers in the interval $0 \leq x \leq 2\pi$. We know that

$$\cos \frac{\pi}{6} = \frac{\sqrt{3}}{2} = 0.866025\ldots$$

x	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{5\pi}{6}$	π	$\frac{7\pi}{6}$	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	$\frac{11\pi}{6}$	2π
cos x	1	0.87	0.5	0	−0.5	−0.87	−1	−0.87	−0.5	0	0.5	0.87	1



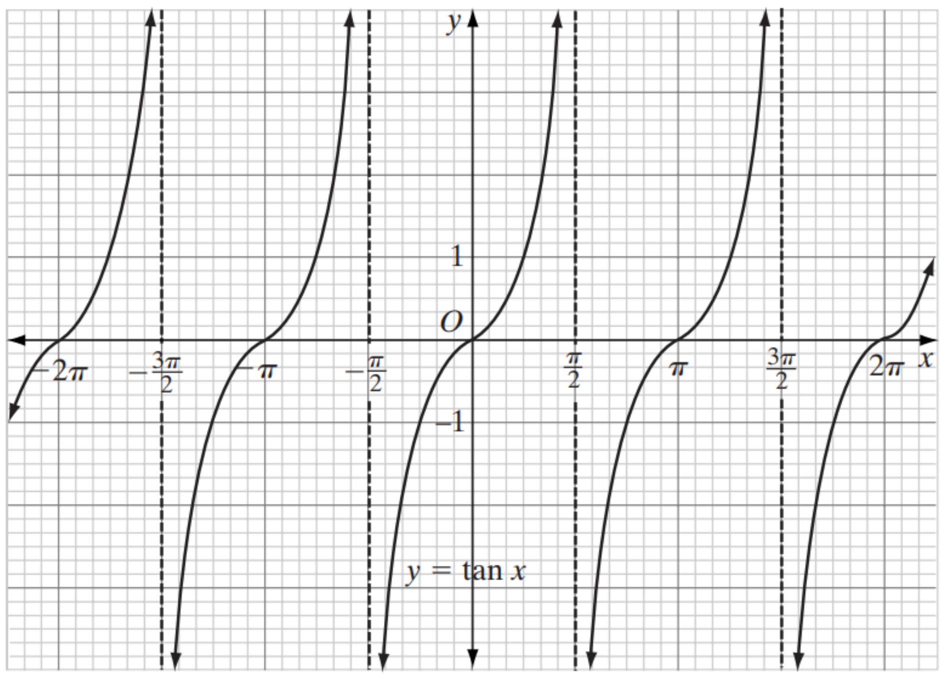
The function $y = \cos x$ is a periodic function with a period of 2π because for every x in the domain of the cosine function, $\cos x = \cos (x + 2\pi)$.

Gragh of The Tangent Function

We can use the table shown below to draw the graph of $y = \tan x$. The values of x are given at intervals of $\frac{\pi}{6}$ from -2π to 2π. The values of $\tan x$ are the approximate decimal values displayed by a calculator, rounded to two decimal places. No value is listed for those values of x for which $\tan x$ is undefined.

x	-2π	$-\frac{11\pi}{6}$	$-\frac{5\pi}{3}$	$-\frac{3\pi}{2}$	$-\frac{4\pi}{3}$	$-\frac{7\pi}{6}$	$-\pi$	$-\frac{5\pi}{6}$	$-\frac{2\pi}{3}$	$-\frac{\pi}{2}$	$-\frac{\pi}{3}$	$-\frac{\pi}{6}$
tan x	0	0.58	1.73	—	−1.73	−0.58	0	0.58	1.73	—	−1.73	−0.58

x	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{5\pi}{6}$	π	$\frac{7\pi}{6}$	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	$\frac{11\pi}{6}$	2π
tan x	0	0.58	1.73	—	−1.73	−0.58	0	0.58	1.73	—	−1.73	−0.58	0



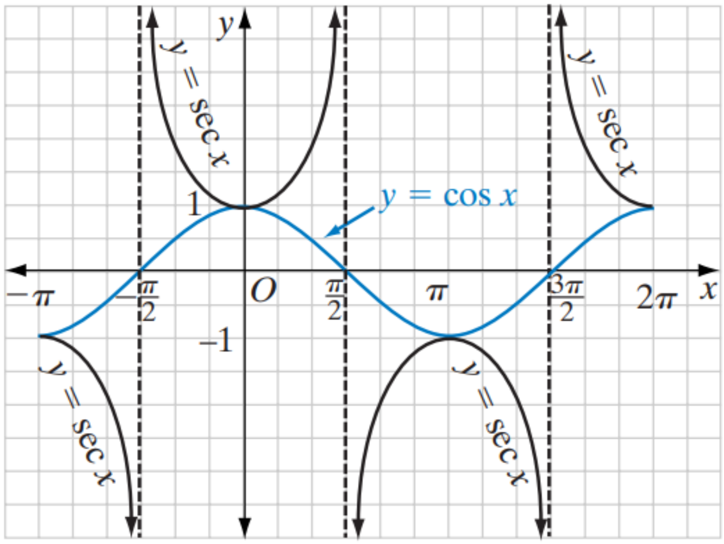
The graph of the tangent function is a curve that increases through negative values of tan x to 0 and then continues to increase through positive values.

Gragh of The Secante Function

The secant function is defined in terms of the cosine function: $\sec x = \frac{1}{\cos x}$. To graph the secant function, we can use the reciprocals of the cosine function values. Reciprocal values of the cosine function exist for $-1 \le \cos x < 0$, and for $0 < \cos x \le 1$. Therefore:

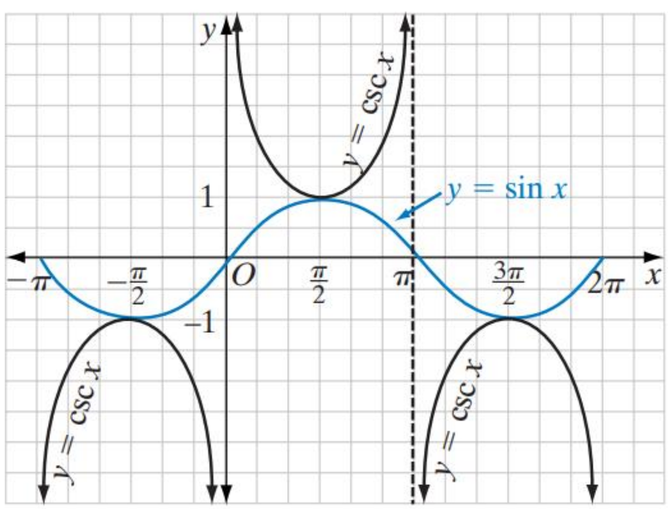
$$-\infty < \sec x \le -1 \qquad 1 \le \sec x < \infty$$

For integral values of n, the vertical lines on the graph at $x = \frac{\pi}{2} + n\pi$ are asymptotes.



Gragh of The Cosecant Function

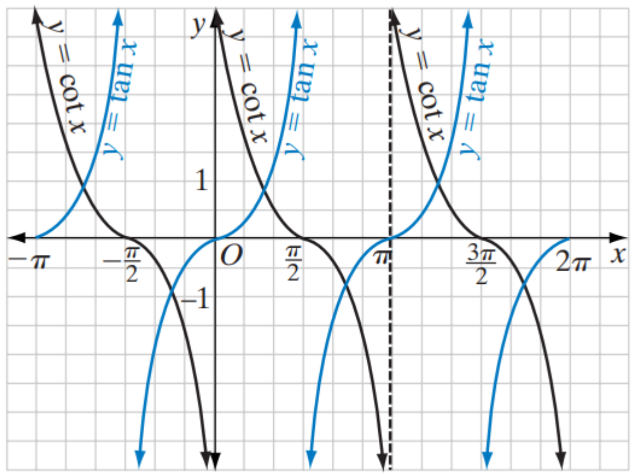
For values of x that are multiples of π, $\sin x = 0$ and $\csc x$ is undefined. For integral values of n, the vertical lines on the graph at $x = n\pi$ are asymptotes.



The cosecant function is defined in terms of the sine function: $\csc x = \frac{1}{\sin x}$. To graph the cosecant function, we can use the reciprocals of the sine function values.

Gragh of The Cotan gent Function

The cotangent function is defined in terms of the tangent function: $\cot x = \frac{1}{\tan x}$. To graph the cotangent function, we can to use the reciprocals of the tangent function values. For values of x that are multiples of π, $\tan x = 0$ and $\cot x$ is undefined. For values of x for which $\tan x$ is undefined, $\cot x = 0$. For integral values of n, the vertical lines on the graph at $x = n\pi$ are asymptotes.



INVERSE TRIGOMETRIC FUNCTIONS

DEF.: The inverse sine fun. Denoted by $\sin^{-1} x$ is defined to be the inverse of restricted sine fun. $\sin x \quad -\frac{\pi}{2} \le x \le \frac{\pi}{2}$

DEF.: The inverse cosine fun. Denoted by $\cos^{-1} x$ is defined to be the inverse of restricted cosine fun. $\cos x \quad 0 \le x \le \pi$

DEF.: The inverse tangent fun. Denoted by $\tan^{-1} x$ is defined to be the inverse of restricted tangent fun. $\tan x \quad -\frac{\pi}{2} \le x \le \frac{\pi}{2}$

DEF.: The inverse secant fun. Denoted by $\sec^{-1} x$ is defined to be the inverse of restricted secant fun. $\sec x \quad 0 \le x \le \pi \;, x \ne \frac{\pi}{2}$

<u>**NOTE**</u> : $\sin^{-1} x \ne \frac{1}{\sin x}$

EXAM :

$$(1)\ if\ \ x=\sin^{-1}\frac{1}{2}\ ,\ find\ value\ \ of\ \ x$$

$$x=\sin^{-1}\frac{1}{2}\Rightarrow \sin x=\cancel{\sin}\ \cancel{\sin^{-1}}\frac{1}{2}\Rightarrow \sin x=\frac{1}{2}\Rightarrow x=\frac{\pi}{6}$$

$$(2)\ simplify\quad \sin(\sin^{-1}\frac{1}{2})$$

$$\because \sin^{-1}\frac{1}{2}=\frac{\pi}{6}\Rightarrow \sin(\sin^{-1}\frac{1}{2})=\sin(\frac{\pi}{6})=\frac{1}{2}$$

$$(3)\ show\ \ that\quad \cos(\sin^{-1}\frac{1}{2})=\frac{\sqrt{3}}{2}$$

$$\because \cos x=\sqrt{1-\sin^2 x}$$

$$\therefore \cos(\sin^{-1}x)=\sqrt{1-\sin^2\sin^{-1}\frac{1}{2}}=\sqrt{1-\sin^2\frac{\pi}{6}}=\sqrt{1-(\frac{1}{2})^2}=\frac{\sqrt{3}}{2}$$

مشتقات الدوال المثلثية العكسية :

$$\boxed{1}\ \frac{d}{dx}\sin^{-1}u=\frac{1}{\sqrt{1-u^2}}\frac{du}{dx}$$

$$\boxed{2}\ \frac{d}{dx}\cos^{-1}u=\frac{-1}{\sqrt{1-u^2}}\frac{du}{dx}$$

$$\boxed{3}\ \frac{d}{dx}\tan^{-1}u=\frac{1}{1+u^2}\frac{du}{dx}$$

$$\boxed{4}\ \frac{d}{dx}\cot^{-1}u=\frac{-1}{1+u^2}\frac{du}{dx}$$

$$\boxed{5}\ \frac{d}{dx}\sec^{-1}u=\frac{1}{|u|\sqrt{u^2-1}}\frac{du}{dx}$$

$$\boxed{6}\ \frac{d}{dx}\csc^{-1}u=\frac{-1}{|u|\sqrt{u^2-1}}\frac{du}{dx}$$

EXAM : **Find** $\frac{dy}{dx}$ **to :**

$$y = \sin^{-1} 2x \Rightarrow \frac{dy}{dx} = \frac{2}{\sqrt{1-4x^2}}$$

$$y = \tan^{-1} 3x + e^{\tan^{-1} x} \Rightarrow \frac{dy}{dx} = \frac{3}{1+9x^2} + e^{\tan^{-1} x} \cdot \frac{1}{1+x^2}$$

$$y = \cos^{-1} \cos x \Rightarrow \frac{dy}{dx} = \frac{-(-\sin x)}{\sqrt{1-\cos^2 x}} = \frac{\sin x}{\sin x} = 1$$

$$OR \quad \cancel{\cos^{-1}} \cancel{\cos} x = x \Rightarrow \frac{dy}{dx} = 1$$

$$y = e^x \sec^{-1} x \Rightarrow \frac{dy}{dx} = e^x \cdot \frac{1}{x\sqrt{x^2-1}} + \sec^{-1} x \cdot e^x$$

Homework

1) Find $\frac{dy}{dx}$ to

$$y = Ln(\cos^{-1} x) \quad , \quad y = \sqrt{\cot^{-1} x} \quad ,$$

$$y = (\tan x)^{-1} \quad , \qquad y = \cot^{-1} \sqrt{x}$$

2) Find $\frac{dy}{dx}$ to

$$x^3 + x \tan^{-1} y = e^y \qquad \sin^{-1}(xy) = cos^{-1}(x - y)$$

Hyperbolic Functions