المحاضرة الثالثة

Relations and function

1- Cartesian products of sets

Definition: Given two non-empty sets A & B the set of all ordered pairs (x,y), wher $x \in A$ $y \in B$ is called cartesian product of A and B

$$A \times B = \{(x,y) \mid x \in A \text{ and } y \in B\}$$

If $A = \{1,2,3\}$ and $B = \{4,5\}$, then

$A \times B = \{(1,4)(1,5),(2,4),(2,5),(3,4),(3,5)\}$

$B \times A = \{ \cancel{(4,1),(5,1),(4,2)(5,2)}$

$= \{(4,1),(4,2)(4,3),(5,1),(5,2),(5,3)\}$

2. Relations:

A Relation R from anon-empty set A to anon-empty set B is asubset of the cartesian product set $A \times B$

If $R = \{(x,y) \mid x = 2y \ , x \in A, y \in B\}$

$R = \{4, 2\}$

Domain: The set of all first elements in arelation R

Range: The set of all second elements in R

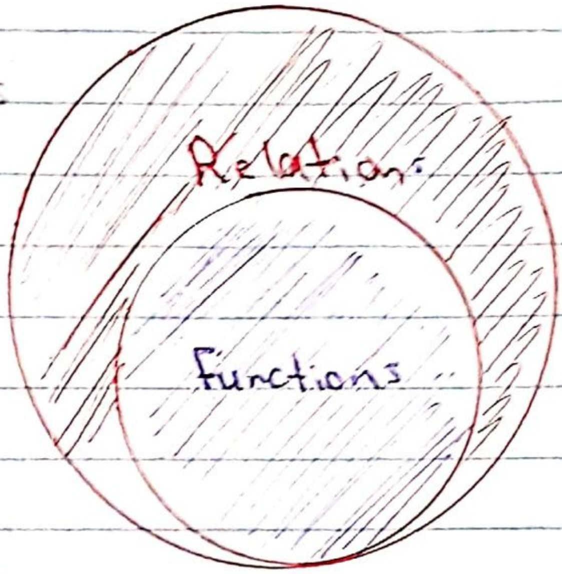
Example:- the set $R = \{(1,2),(-2,3),(\frac{1}{2},3)\}$ is arelation

$D = \{1,-2,\frac{1}{2}\}$. $R = \{2,3\}$

3. Functions: A relation f from a set A to set B is said to be function if every element of a set A has one and only one image in a set B.

$f: X \to Y$ means that f is a function from X to Y
X is call domain and Y is called Co-domain

Note: all functions are relations, but not all relations are functions



Types of functions.

1. one to one or injective function إقتبانية

a function $f: X \to Y$ is said to be one to one if for each element of X there is a distinct element of Y

2. onto function or Surjective function
$f: X \to Y$ is said to be onto for every element of set Y there is a Pre-image in set X.

3 - Bijective function
A function that is both injective and surjective

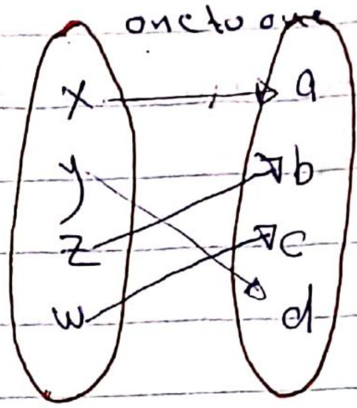
$\forall\, x, y \in A$

$f(x) = f(y) \Rightarrow x = y$ or 1-1

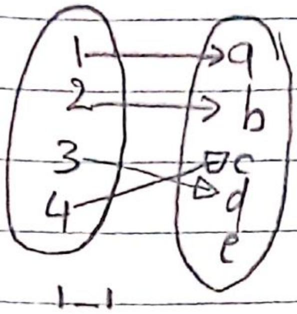
or

$f^{-1}(y) = x$ iff $f(x) = y$

one to one

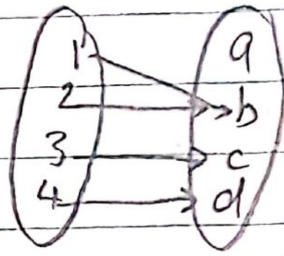


set A B



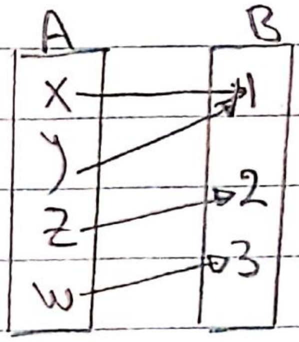
1-1

and

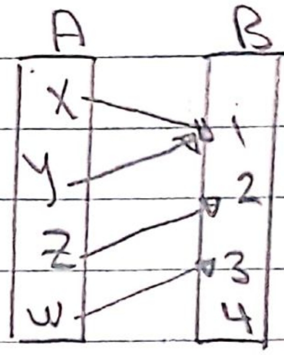


is not an injective.

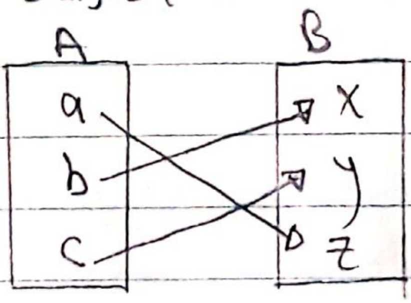
on to function



onto - surjection



Not a surjection



some specific types of function

1. Identity function,

let $f: R \to R$ defined by $y = f(x)$, for each $x \in R$ is called the Identity function

$D_f = R$

$R_f = R$

2. Constant function

$f: R \to R$ defined by $f(x) = C$, $x \in R$, C is constant $C \in R$.

$D_f = R$

$R_f = \{c\}$

3. Polynomial function: A Real valued function $f: R \to R$ defined by

$y = f(x) = a_0 + a_1 x + a_2 x^2 + \dots + a_n x^n$, $n \in N$,
$a_0, a_1, \dots, a_n \in R$, $\forall x \in R$

4. Rational function

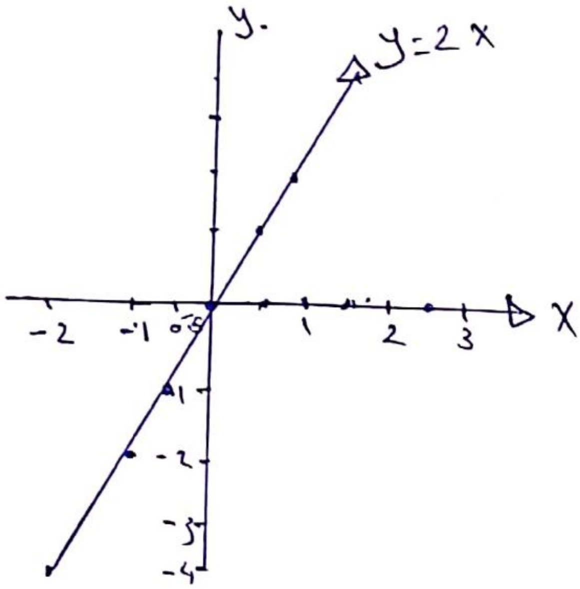
there are the real function of type $\frac{f(x)}{g(x)}$
where $f(x)$, $g(x)$ are polynomaial function and $g(x) \neq 0$

Ex $f: R - \{-2\} \to R$ and $f(x) = \frac{x+1}{x+2}$

$\forall x \in R - \{-2\}$ is arational number

Ex(1) Find the domain and Rang of $y = 2x$

x	-2	-1	$-\frac{1}{2}$	0	$1/2$	1	2
y	-4	-2	-1	0	1	2	4

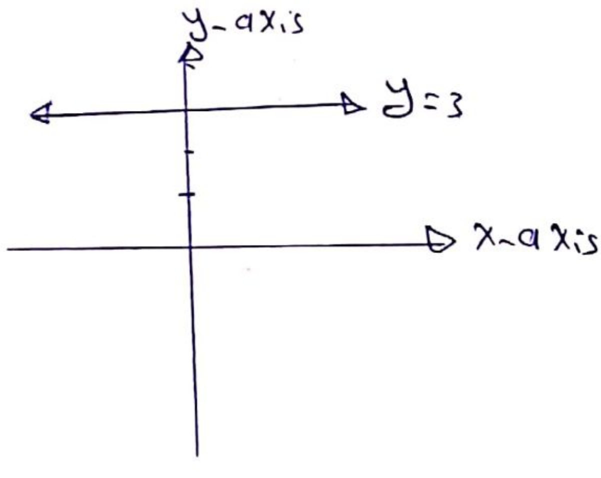


$D_f = R$

$R_f = R$

EX(2): $y = 3$, find D_f and R_f

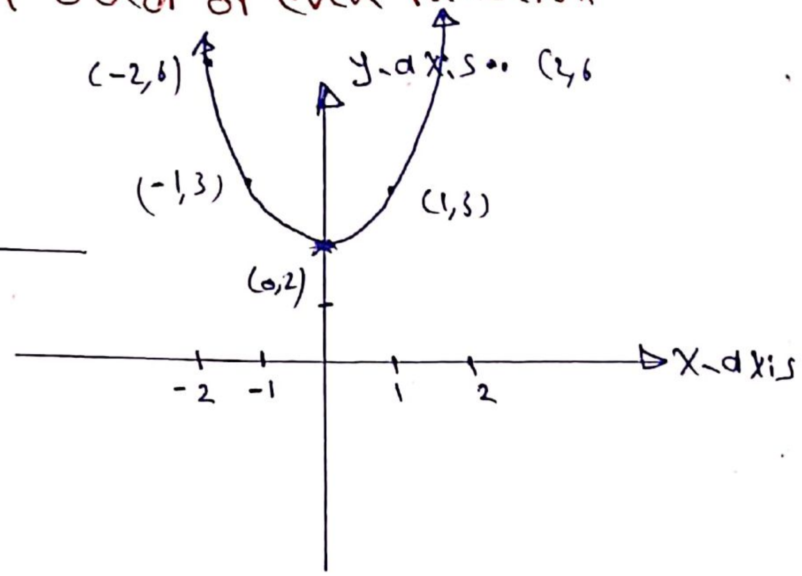
x	-2	-1	0	1	2
y	3	3	3	3	3



$D_f = R$

$R_f = \{3\}$

3) Find D_f and R_f for $y = X^2+2$ and sketch the graph and Is it odd or even function



x	-2	-1	0	1	2
y	6	3	2	3	6

$D_f = R$

$\Rightarrow y = X^2+2$

$\Rightarrow X^2 = y-2$

$X = \mp\sqrt{y-1} \quad \Rightarrow \quad y-1 \geq 0 \quad \Rightarrow y \geq 1$

$\therefore R_f = \{y : y \geq 1$ or $1 \leq y < \infty$ or $[1, \infty]$

$f(-X) = (-X)^2+2 = X^2+2$

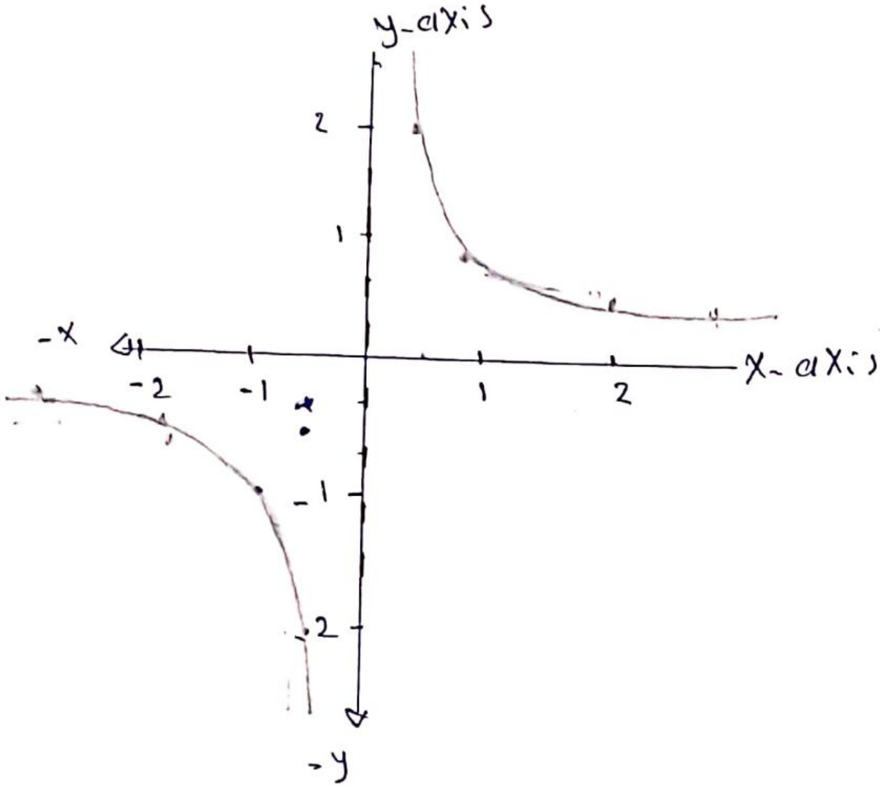
$\therefore f(-X) = f(x) \quad \therefore$ the function is even

Ex( ): Find D_f and R_f for $y = \frac{1}{x}$ and sketch its graph

solution

$x \neq 0 \Rightarrow D_f = R - [0] = (-\infty, 0) \cup (0, \infty)$

x	y
-3	$-\frac{1}{3}$
-2	$-\frac{1}{2}$
-1	-1
$-\frac{1}{2}$	-2
0	نقطة انفصال
$\frac{1}{2}$	2
1	1
2	1/2
3	1/3



$x = 0$

نقطة انفصال

5- the Modulus function: the real function

$f: R \to R$

$$f(x) = |x| = \begin{cases} x , & x \geq 0 \\ -x , & x < 0 \end{cases}$$

$D_f = R$

$R_f = R^+ \cup \{0\} = y \geq 0 = [0, \infty)$

6- Signum function: the real function

$$f: R \to R \quad , \quad f(x) = \begin{cases} \frac{|x|}{x} & x \neq 0 \\ 0 & \text{if } x = 0 \end{cases} = \begin{cases} 1 & \text{if } x > 0 \\ 0 & \text{if } x = 0 \\ -1 & \text{if } x < 0 \end{cases}$$

$D_f = R$

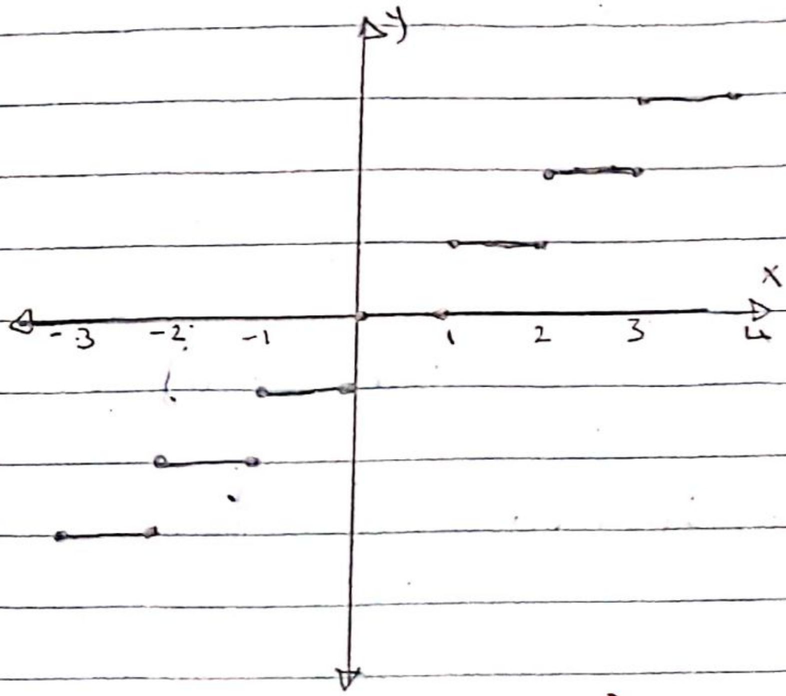
$R_f = \{1, 0, -1\}$

7- Greatest integer function $f: R \to R$

$f(x) = [x] , x \in R$

$$\begin{aligned} f(x) = [x] &= -1 & -1 \leq x < 0 \\ = [x] &= 0 & 0 \leq x < 1 \\ = [x] &= 1 & 1 \leq x < 2 \\ [x] &= 2 & 2 \leq x < 1 \end{aligned}$$

The graph is not continuous



Example:- Find the greatest integer function for the following.

a) $[-121]$ ans $= -121$

b) $[3.501]$ ans $= 3$

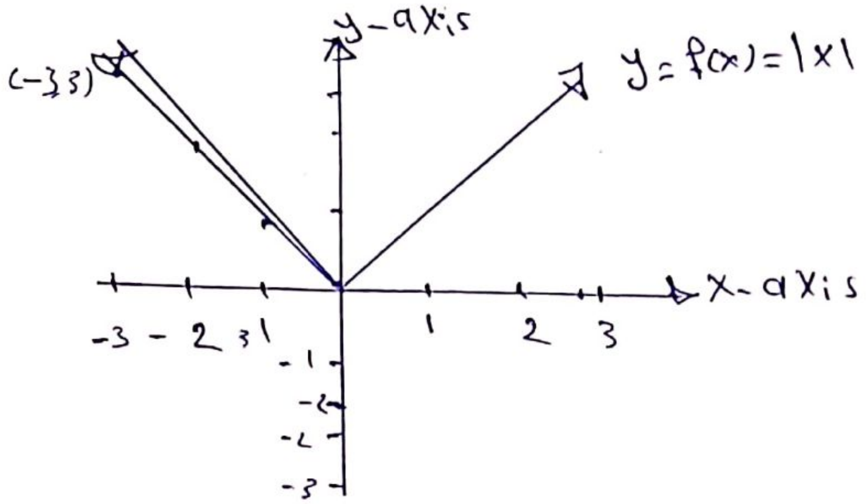
c) $[-1.898]$ ans $= -2$

$D_f = \mathbb{R}$
$R_f = \mathbb{Z}$

Ex: Find the D_f and R_f for $y = |x|$ and sketch the graph and Is the function is odd or even.

$$f(x) = |x| = \begin{cases} x & x \geq 0 \\ -x & x < 0 \end{cases}$$

x	y
-3	3
-2	2
-1	1
0	0
1	1
2	2
3	3



$D_f = \mathbb{R}$

$R_f = y \geq 0 \Rightarrow R_f = [0, \infty)$

$f(-x) = |-x| = |x| = f(x)$

$\because f(-x) = f(x) \quad \therefore f$ is even

Example: (Homeworks). Sketch the graph to the function

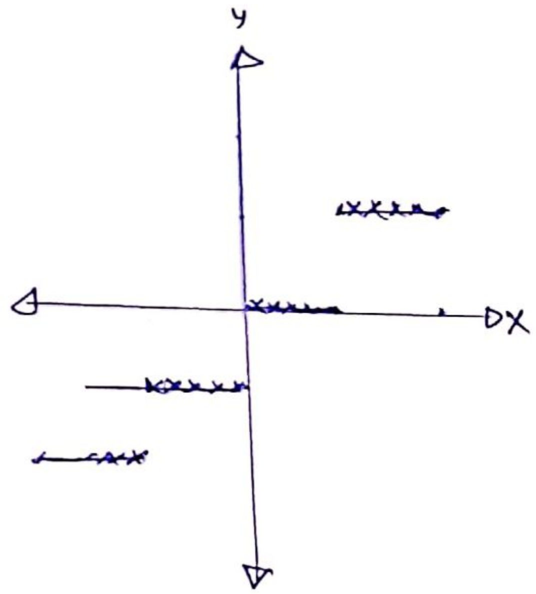
$y=|x+1|$, $y=|x|+1$, $y=|x|-1$, $y=|x-1|$

$y=1+|x|$, $y=|1-x|$, $y=1-|x|$

and find D_f, R_f, Is the function is odd or even?

Ex: Sketch the graph for the function $y=[x]$ and find D_f, R_f

x	$y=[x]$
-1	-1
-0.9	$[-0.9]=-1$
-0.6	$[-0.6]=-1$
-0.5	$[-0.5]=-1$
-0.1	$[-0.1]=-1$
0	$[0]=0$
0.1	$[0.1]=0$
0.5	$[0.5]=0$
0.6	$[0.6]=0$
0.9	$[0.9]=0$
1	$[1]=1$
1.1	$[1.1]=1$
1.5	$[1.5]=1$
1.9	$[1.9]=1$
2	$[2]=2$
2.1	$[2.1]=2$
2.5	$[2.5]=2$



$D_f = R$

$R_f = \{0, \mp 1, \mp 2, \mp 3, \cdots\} = Z$

Algebra of real function العمليات الجبرية على الدوال

1) Addition of two real function
let $f: X \to R$, and $g: X \to R$ be any two real function, where $X \subseteq R$ then we define $(f+g): X \to R$
$(f+g)(x) = f(x) + g(x)$, $\forall x \in R$

2) Subtraction of a real function from another
let $f: X \to R$ and $g: X \to R$ be any two real function, where $X \subseteq R$, then, we define
$(f-g): X \to R$
$(f-g)(x) = f(x) - g(x) \quad \forall x \in R$

3) multiplication by a scalar
Let $f: X \to R$ be a real function, where $X \subseteq R$
then the product $cf: X \to R$ defined by

$(cf)(x) = c\,f(x) \quad \forall x \in X$

4) Multiplication two function
let $f: X \to R$, $g: X \to R$ be any two real function, where $X \subseteq R$. then the product $f.g: X \to R$ defined by
$(fg)(x) = f(x).g(x) \quad \forall x \in X$

5) Quotient of two real functions حاصل قسمة دالتين
let f and g be two real function defined from $X \to R$

the quotient of f by g, $\frac{f}{g} : X \to R$ as

$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}, \quad g(x) \neq 0, \; x \in X$$

Note: Domain of sum function $f+g$, difference function $f-g$, and Product function $f.g$
$= \{X : X \in D_f \cap D_g\}$

where –

D_f = domain of function f
D_g = domain of function g

Domain of quotient function $\frac{f}{g}$ =

$$D_{\frac{f}{g}} = \{X : X \in D_f \cap D_g \text{ and } g(x) \neq 0\}$$