

(4) المراجعة

Domain and Range

Rules of Finding Domain of a Function

1) Polynomial (linear, quadratic, cubic, etc) function $\mathbb{R}$

2) square root function $\sqrt{x}$, $x \geq 0$

3) exponential function a^x , $\mathbb{R}$

4) Logarithmic function $\log_a(x)$, $x > 0$

5) Rational function $h(x) = \frac{f(x)}{g(x)}$, $g(x) \neq 0$

Example(1): Find The Domain of the following function

1. $f(x) = \sqrt{x+3}$ $x+3 \geq 0 \Rightarrow x \geq -3$, $D_f = [-3, \infty)$

2. $g(x) = \frac{2x+1}{(x-2)}$ $(x-2) \neq 0 \Rightarrow x \neq 2$

the domain is the set of all real number except 2

D_g is $(-\infty, 2) \cup (2, \infty)$

Rules of Finding Range of a Function

1) Linear Function is $\mathbb{R}$

2) Quadratic Function $y = a(x-h)^2 + k$ is

1) $y \geq k$ if $a > 0$

2) $y \leq k$ if $a < 0$

3) Square root function $y \geq 0$

4) exponential function $y > 0$

5) Logarithmic Function $\mathbb{R}$

6) Rational Function, Solve it for x
and set the denominator $\neq 0$
مثلاً

Example. Find the domain and Range

$X = \{1, 2, 3, 4, 5\}$ and $Y = \{1, 2, 3, 4, 5, 6\}$
and $f: X \rightarrow Y$, where $R = \{(x, y) : y = x + 1\}$

Domain = $\{x\} = \{1, 2, 3, 4, 5\}$ = input value

Range = $\{1+1, 2+1, 3+1, 4+1, 5+1\} = \{2, 3, 4, 5, 6\}$

Y is called codomain but not Range.

The Graph of a function

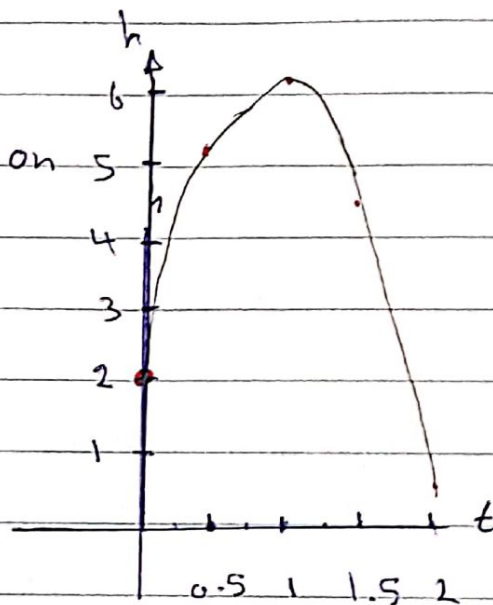
The graph of a function is the set of all points whose coordinates (x, y) satisfy the function $y = f(x)$, this means that for each x -value which is obtained when we substitute into the expression for $f(x)$

- 1- select a few values of x (at least 5)
- 2- obtain the corresponding values of the function
- 3- plot these points

Example 1: Graph the function

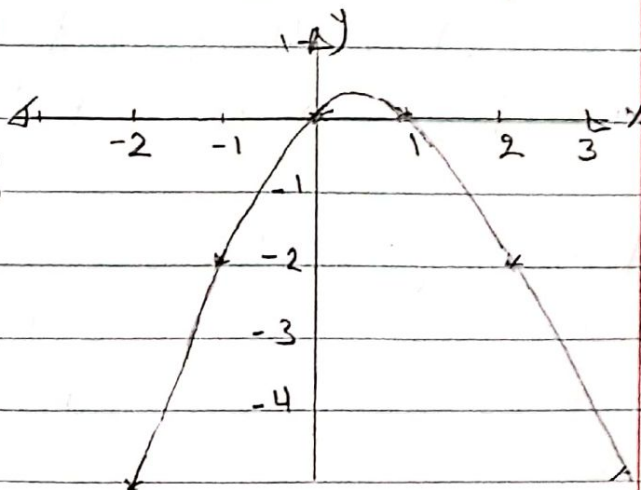
$$h(t) = 2 + 9t - 4.9t^2$$

t	0	0.5	1	1.5	2
h	2	5.3	6.1	4.5	0.4



Example 2: Graph the function $y = x - x^2$

x	-2	-1	0	1	2	3
y	-6	-2	0	0	-2	-6



Example 3: Graph the function $y = 1 + \frac{1}{x}$

x	-4	-3	-2	-1	1	2	3	4
y	$\frac{3}{4}$	$\frac{2}{3}$	$\frac{1}{2}$	0	2	$\frac{3}{2}$	$\frac{4}{3}$	$\frac{5}{4}$

y is not defined at $x=0$

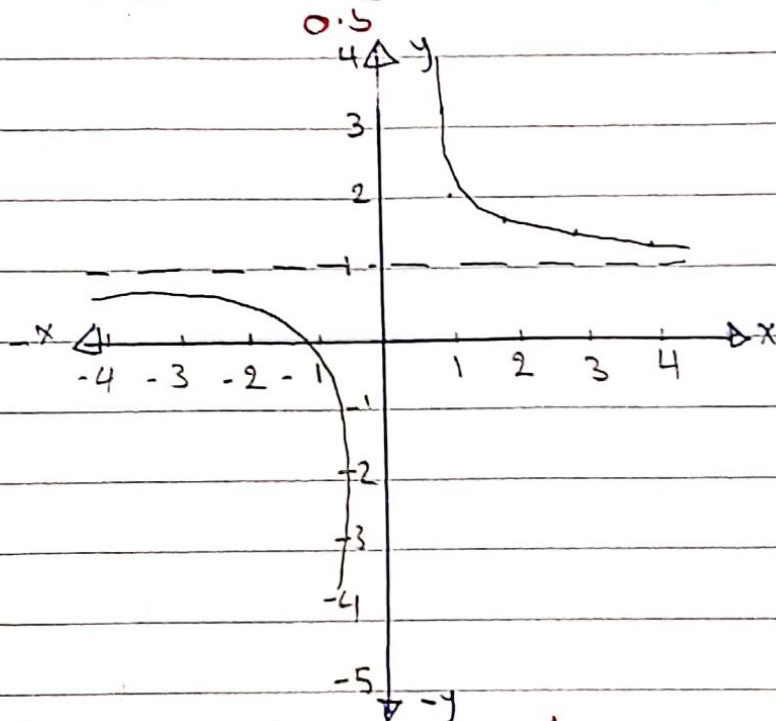
$$x = -0.5 \quad y = 1 + \frac{1}{-0.5} = 1 - 2 = -1$$

عند التقاطع

$$x=0, y=1$$

$$x = 0.5, y = 1 + \frac{1}{0.5} = 1 + 2 = 3$$

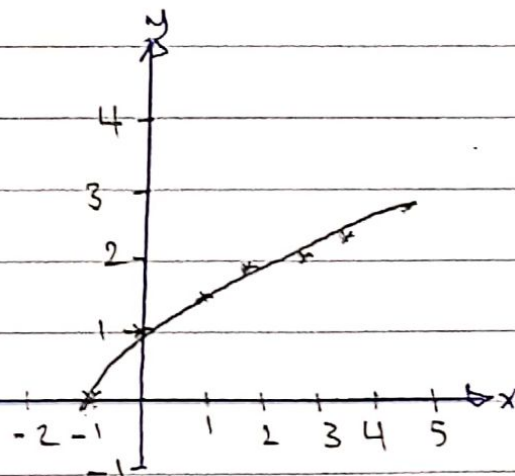
نرسم نقاطي افقي



Example 4: Graph the function $y = \sqrt{y+1}$

م/الدالة لا يكون مرتبة اذا كانت $y < -1$

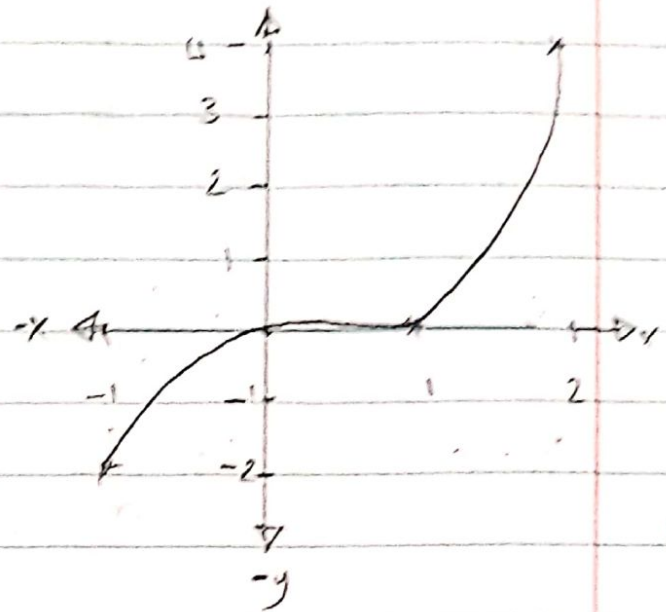
x	-1	0	1	2	3	4	5
y	0	1	1.4	1.7	2	2.2	2.4



EXERCISES : Graph the Functions

1- $y = x^3 - x^2$

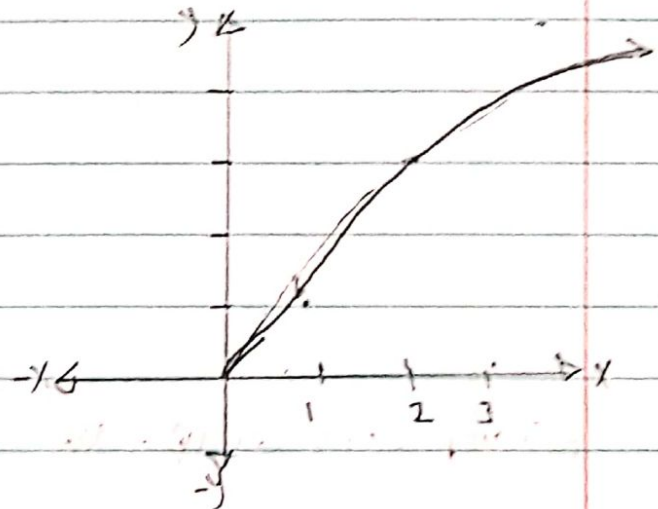
x	-1	0	1	2	3	4
y	-2	0	0	4	18	60



2) $y = \sqrt{x}$

سوف نأخذ القيم الموجبة $x \geq 0$

x	-2	-1	0	1	2	3
y	4	1	1	1	4	9

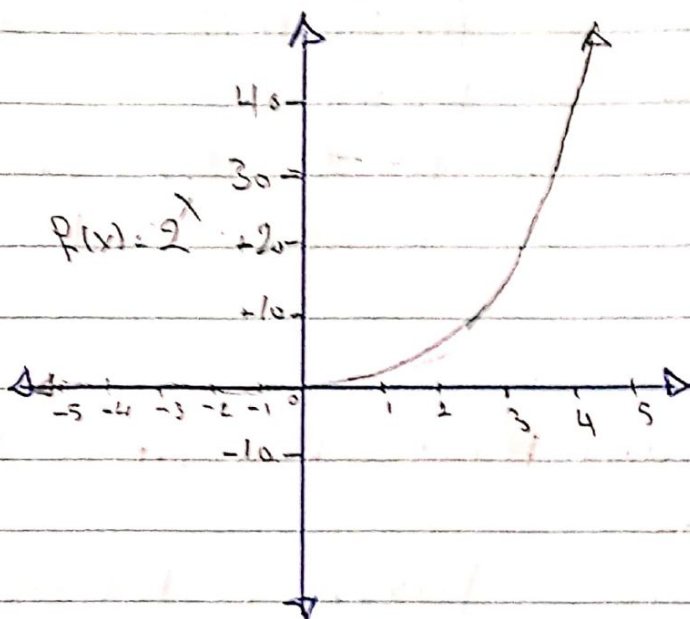


Domain and Range of Exponential Functions

Example: $f(x) = 2^x$

Domain of $f(x) = 2^x$ is $\mathbb{R}$

Range of $f(x)$ is $\{y \in \mathbb{R}, y > 0\}$

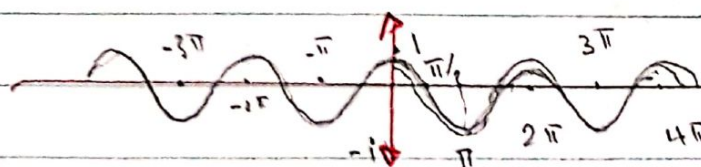
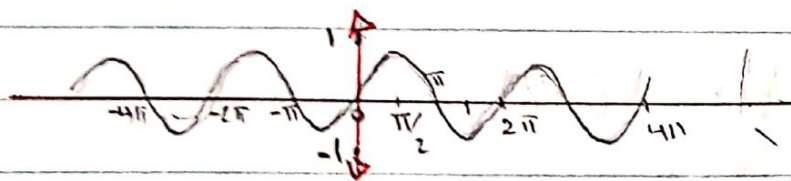


Domain and range of Trigonometric Function

the function $\sin(x)$ and $\cos(x)$

D_f is the set $\mathbb{R}$ or $(-\infty, +\infty)$

R_f is $[-1, 1]$



The domain and range of all trigonometric function are shown below:

Trigonometric Function	Domain	Range
1 - $\sin \theta$	$(-\infty, +\infty)$	$[-1, 1]$
2 - $\cos \theta$	$(-\infty, +\infty)$	$[-1, 1]$
3 - $\tan \theta$	$\mathbb{R} - (2n+1)\pi/2$	$(-\infty, +\infty)$
4 - $\cot \theta$	$\mathbb{R} - n\pi$	$(-\infty, +\infty)$
5 - $\sec \theta$	$\mathbb{R} - (2n+1)\pi/2$	$(-\infty, -1] \cup [1, +\infty)$
6 - $\csc \theta$	$\mathbb{R} - n\pi$	$(-\infty, -1] \cup [1, +\infty)$

Domain and Range of an Absolute value Function.

the function $y = |ax+b|$ is defined for all real numbers

Domain = $\mathbb{R}$

Range = $[0, \infty)$

Example: Find the domain and Range of

$$f(x) = |6-x|$$

$$D_f = \mathbb{R}$$

$$R_f = [0, \infty)$$

Domain and Range of square Root Function

$f(x) = \sqrt{ax+b}$ this function is not defined for negative numbers
this function is defined only when

$$ax+b \geq 0 \Rightarrow ax \geq -b \Rightarrow x \geq -\frac{b}{a}$$

Example: calculate the domain and range of the function $f(x) = 2 - \sqrt{-3x+2}$

Domain: A square root function is defined only when the value inside it is a non-negative number, so for domain

$$-3x+2 \geq 0$$

$$-3x \geq -2 \quad x \leq \frac{2}{3}$$

$$3x \leq 2 \quad \div 3$$

$$x \leq \frac{2}{3}$$

$$D_f = (-\infty, \frac{2}{3}]$$

Range: we already know that the square root function result is a non-negative value always

$$y \geq 0, \text{ let's find } y \text{ for } x$$

$$\sqrt{-3x+2} \geq 0 \quad x \leq \frac{2}{3}$$

$$-\sqrt{-3x+2} \leq 0$$

adding 2 on both sides

$$2 - \sqrt{-3x+2} \leq 2 \neq 0$$

$$y \leq 2$$

$$R_f = (-\infty, 2)$$

افقي
vertical
افقي
horizontal

Inverse Function

Inverse function is represented by f^{-1} and it exists only when f is both one-one and onto function such that the composition of the function f and the inverse of it f^{-1} gives the domain value of x

$$(f \circ f^{-1})(x) = (f^{-1} \circ f)(x)$$

Example: Find the Inverse Function for the given function

$$f(x) = \frac{4x+1}{3x-2}$$

$$y = \frac{4x+1}{3x-2}$$

$$x = \frac{4y+1}{3y-2}$$

$$f^{-1}(x) = \frac{2x+1}{3x-4}$$

الفعل $f(x)$

غير كل x ب y

كل x له y

الفعل $f^{-1}(x)$

$$\Rightarrow 3xy - 2x = 4y + 1$$

$$3xy - 4y = 2x + 1$$

$$y(3x - 4) = 2x + 1$$

$$\therefore y = \frac{2x+1}{3x-4}$$

$$\Leftarrow 3x - 4$$

Example 2: Find the inverse of the function
 $f(x) = 2x + 3$

$$y = 2x + 3$$

$$x = \frac{y - 3}{2}$$

$$\Rightarrow y = \frac{x - 3}{2}$$

١- نضع بدل x في $f(x)$ y

٢- نضع بدل y في $f(y)$

٣- نحل المعادلة بالنسبة لـ y

٤- نضع بدل y في $f(x)$