

distance between two point and between point and line

1) the distance between two point is the length of the line segment that connects the two given point

المسافة بين نقطتين هي طول الخط المستقيم الذي يربط بين النقطتين



The formula for the distance between two points

is where (x_1, y_1) and (x_2, y_2) are coordinates is given by

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

this is also known as the Euclidean Distance Formula to find the distance between the points with coordinates (x_1, y_1, z_1) , (x_2, y_2, z_2) given in 3-D plane as

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

Example: Find the distance between the points with coordinates given as $A = (1, 2)$, $B = (1, 5)$

Solution

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(1-1)^2 + (5-2)^2} = \sqrt{9} = 3 \text{ units}$$

Example 2: Find the distance of the point $P(4,3)$ from the origin.

Solution:

$$P(4,3) = P(x_1, y_1) \quad (x_2, y_2) = (0,0)$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$d = \sqrt{(0-4)^2 + (0-3)^2} = \sqrt{16+9} = \sqrt{25} = 5 \text{ units}$$

Example: Determine whether the triangle with vertices $A(-1,2)$, $B(4,7)$, $C(-3,6)$ is isosceles

Sol

متساوي الاضلاع

$$\overline{AB} = \sqrt{(-1-4)^2 + (2-7)^2} = \sqrt{25+25} = \sqrt{50}$$

$$\overline{AC} = \sqrt{(-1+3)^2 + (2-6)^2} = \sqrt{4+16} = \sqrt{20}$$

$$\overline{BC} = \sqrt{(4+3)^2 + (7-6)^2} = \sqrt{49+1} = \sqrt{50}$$

Since $\overline{AB} = \overline{BC}$ the triangle is isosceles.

∵ ضالتي ضلعتين متساويتين في المثلث اذن المثلث متساوي الاضلاع.

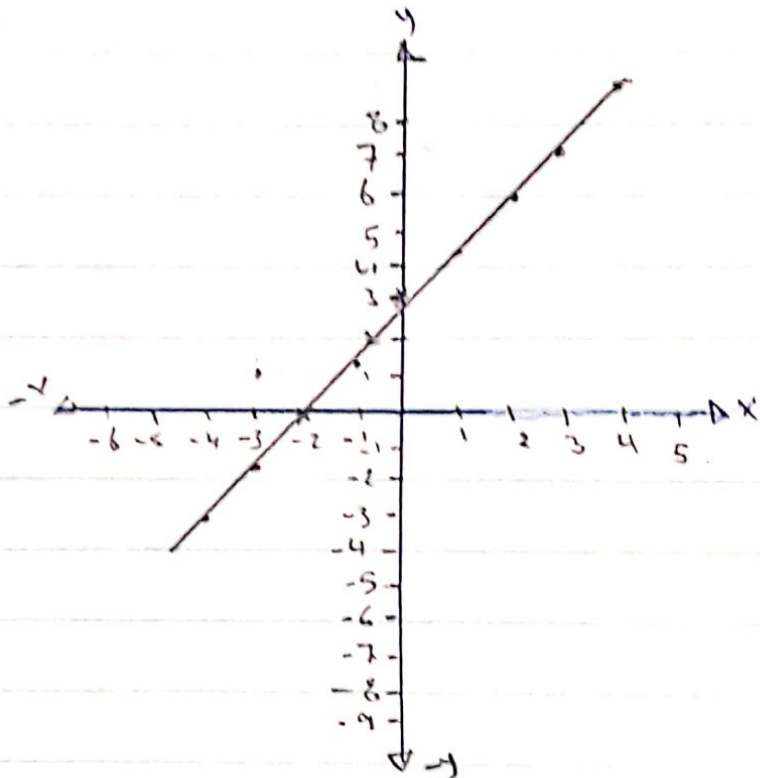
Graph of Equations

Consider the following Equation in Volving the Variable x and y .

$$2y - 3x = 6 \rightarrow 2y = 6 + 3x \rightarrow y = \frac{6 + 3x}{2}$$

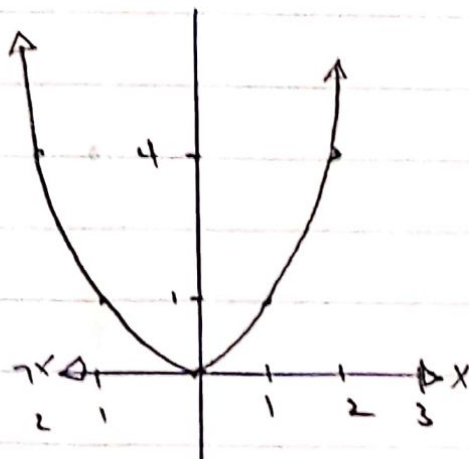
$$y = \frac{6 + 3x}{2}$$

x	y
4	9
3	$15/2$
2	6
1	$9/2$
0	3
-1	$3/2$
-2	0
-3	$-3/2$
-4	-3



Example: Graph $y = x^2$

x	y^2
3	9
2	4
1	1
0	0
-1	1
-2	4



Straight lines and their equation

The slope of the line through the points (x_1, y_1) (x_2, y_2) is

$$\text{slope} = m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\text{change in } y}{\text{change in } x} = \frac{\Delta y}{\Delta x}$$

Example: Find the slope of the line L through the point $P_1 = (1, -1)$ and $P_2 = (4, 2)$.

Solution:

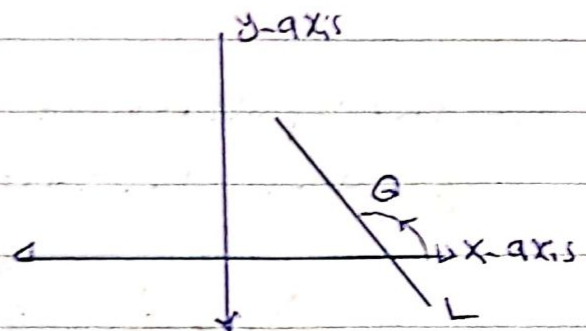
$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2 - (-1)}{4 - 1} = \frac{3}{3} = 1$$

Inclination زاوية الميل

The smallest angle θ between the positive x -axis and the line L , measured from the x -axis to L in the counter clockwise direction.

هي الزاوية التي راسها في الأصل ونقيص مع الاتجاه الموجب محور السينات

the slope $m = \tan \theta$



مثال: Find the inclination of the line through the points $(0, -1)$, $(\sqrt{3}, 0)$

Solution:

$$\text{slope } m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0 - (-1)}{\sqrt{3} - 0} = \frac{1}{\sqrt{3}}$$

$$\tan \theta = \frac{1}{\sqrt{3}}, \quad \theta = 15^\circ$$

1. the equation of the line with slope m and y -intercept b is

$$y = mx + b \quad b = \text{y-intercept} \quad m = \text{slope}$$

معادلة الخط إذا كان ميله معلوم، ونقطه المحور y

2. the equation of the line with slope m and going through a given point $P_1 = (x_1, y_1)$ is

معادلة الخط إذا كان ميله معلوم، ونقطه

$$y - y_1 = m(x - x_1)$$

3. the equation of the line with slope m and going through the origin $(0, 0)$ is

$$y = mx$$

4. the equation of the line with x -intercept a and y -intercept b is

$$\frac{x}{a} + \frac{y}{b} = 1 \quad \text{if } a \neq 0, b \neq 0$$

Ex: Find the equation of the line going through the point $(5,3), (-1,6)$.

Sol.:

$$\text{slope} = m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{6-3}{-1-5} = \frac{-3}{6} = -\frac{1}{2}$$

the eq.

$$y - y_1 = m(x - x_1)$$

$$y - 3 = -\frac{1}{2}(x - 5)$$

$$y - 3 = -\frac{1}{2}x + \frac{5}{2}$$

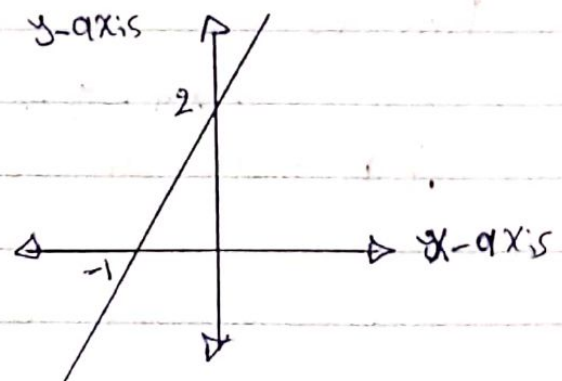
$$y = -\frac{1}{2}x + \frac{11}{2}$$

Ex: Find the eq. of the line with the given x -intercept (-1) and y -intercept (2)

Sol.: $\frac{x}{a} + \frac{y}{b} = 1 \rightarrow \frac{x}{-1} + \frac{y}{2} = 1$

$$\frac{2x - y}{-2} = 1 \rightarrow 2x - y = -2$$

$$y = 2x + 2$$



Ex: Graph the equation $3x + 2y - 6 = 0$

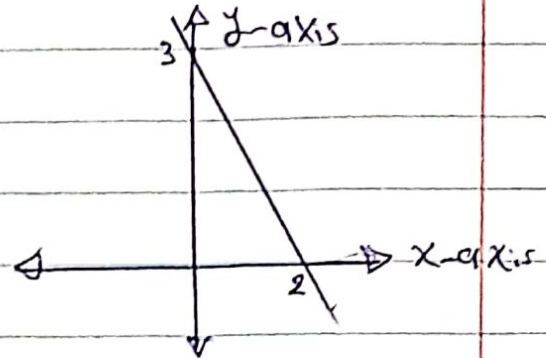
Sol:

$$\text{at } x=0 : 3(0) + 2y = 6 \Rightarrow y=3 \quad (0, 3)$$

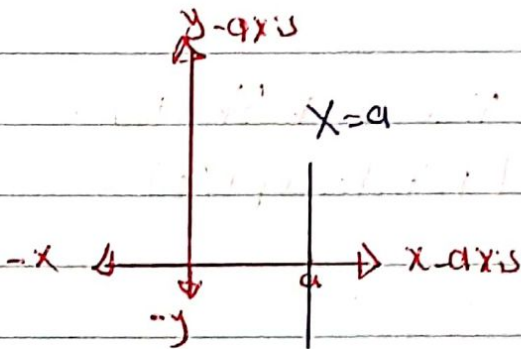
$$\text{at } y=0 : 3x + 2(0) = 6 \Rightarrow x=2 \quad (2, 0)$$

x-intercept = 2

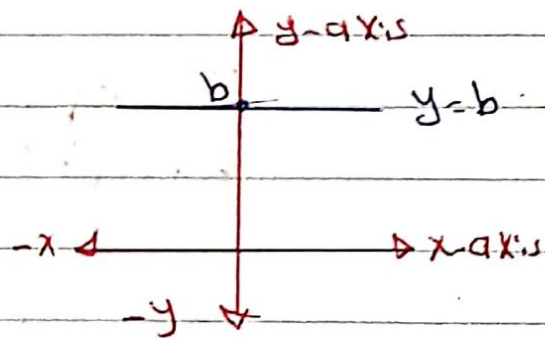
y-intercept = 3



the equation of a vertical straight line is $x=a$



the equation of the horizontal straight line is $y=b$



The general formula of the equation of the line is

$$Ax + By + C = 0$$

A, B and C are constant

$$\text{the slope of the line} = \frac{-A}{B} = \frac{\text{مقابل } x}{\text{مقابل } y}$$

parallel lines

المستقيمت المتوازية

$$L_1 \parallel L_2 \rightarrow m_1 = m_2$$

لتوازي مستقيمان اذا تساوى ميلهما

perpendicular lines

المستقيمت المتعامدة

$$L_1 \perp L_2 \rightarrow m_1 \cdot m_2 = -1$$

two lines are perpendicular iff the product of their slope is (-1)

H.w.1 - Find the equation of the line whose x-intercept $(-\frac{3}{5})$ and y-intercept (2)

2. Find the angle between the two lines
 $3x - 2y + 7 = 0$ and $2x + 3y - 3 = 0$