EQUATION OF STRAIGHT LINE : معادلـــة الخـط المستـقـيـم

The general form of straight line equation is :-

$$ax + by + c = 0$$

OR

$$y = mx + b$$

EXAM : - Find the equation of the curve whose slope at any point p(x,y) is 2x+1 and passing through the point (1,3) .

$$m = \frac{dy}{dx} = 2x + 1$$

$$y = \int \frac{dy}{dx} dx = \int (2x + 1) dx \Rightarrow \boxed{y = x^2 + x + C} \Leftarrow \text{general curves}$$

$$(1,3) \in curves \Rightarrow 3 = 1^2 + 1 + C \Rightarrow C = 1$$

$$\boxed{y = x^2 + x + 1} \Leftarrow \text{special curve}$$

EXAM : - Find the equation of the curve whose slope at any point p(x,y) is $4x^3 + 18x^2 + 8x + 3$ **and passing through the point (1,11) .**

$$m = \frac{dy}{dx} = 4x^3 + 18x^2 + 8x + 3$$

$$y = \int (4x^3 + 18x^2 + 8x + 3) dx$$

$$y = x^4 + 6x^3 + 4x^2 + 3x + C$$

$$11 = 1 + 6 + 4 + 3 + C \Rightarrow C = -3$$

$$y = x^4 + 6x^3 + 4x^2 + 3x - 3$$

DOUBLE INTEGRATION :

EXAM :

$$\int_0^1 \int_0^x (3 - x - y) dy dx$$

$$= \int_0^1 \left. 3y - xy - \frac{1}{2} y^2 \right|_0^x dx = \int_0^1 (3x - x^2 - \frac{1}{2} x^2) dx = \int_0^1 (3x - \frac{3}{2} x^2) dx$$

$$= \left. \frac{3}{2} x^2 - \frac{1}{2} x^3 \right|_0^1 = 1$$

EXAM :

$$\int_{1}^{2}\int_{y}^{y^2} dx\,dy$$

$$=\int_{1}^{2} x\Big|_{y}^{y^2} dy = \int_{1}^{2}(y^2-y)dy = \frac{1}{3}y^3-\frac{1}{2}y^2\Big|_{1}^{2} = \frac{5}{6}$$

EXAM :

$$\int_{0}^{\sqrt{2}}\int_{-\sqrt{4-2y^2}}^{\sqrt{4-2y^2}} y\,dx\,dy$$

$$=\int_{0}^{\sqrt{2}} yx\Big|_{-\sqrt{4-2y^2}}^{\sqrt{4-2y^2}} dx = \int_{0}^{\sqrt{2}} y\sqrt{4-2y^2}+y\sqrt{4-2y^2}dx = 2\int_{0}^{\sqrt{2}} y\sqrt{4-2y^2}dx$$

$$\frac{2}{-4}\frac{(4-2y^2)^{\frac{3}{2}}}{\frac{3}{2}}\Bigg|_{0}^{\sqrt{2}} = \frac{8}{3}$$

Derivatives	**Integrals (Anti derivatives)**
$\frac{d}{dx}(\sin x)=\cos x$;	$\int \cos x\,dx=\sin x+C$
$\frac{d}{dx}(-\cos x)=\sin x$;	$\int \sin x\,dx=-\cos x+C$
$\frac{d}{dx}(\tan x)=\sec^2 x$;	$\int \sec^2 x\,dx=\tan x+C$
$\frac{d}{dx}(-\cot x)=\operatorname{cosec}^2 x$;	$\int \operatorname{cosec}^2 x\,dx=-\cot x+C$

$\frac{d}{dx}(\tan x) = \sec^2 x$; $\int \sec^2 x\, dx = \tan x + C$

$\frac{d}{dx}(-\cot x) = \operatorname{cosec}^2 x$; $\int \operatorname{cosec}^2 x\, dx = -\cot x + C$

$\frac{d}{dx}(\sec x) = \sec x \tan x$; $\int \sec x \tan x\, dx = \sec x + C$

$\frac{d}{dx}(-\operatorname{cosec} x) = \operatorname{cosec} x \cot x$; $\int \operatorname{cosec} x \cot x\, dx = -\operatorname{cosec} x + C$

$\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}$; $\int \frac{dx}{\sqrt{1-x^2}} = \sin^{-1} x + C$

$\frac{d}{dx}(-\cos^{-1} x) = \frac{1}{\sqrt{1-x^2}}$; $\int \frac{dx}{\sqrt{1-x^2}} = -\cos^{-1} x + C$

$\frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2}$; $\int \frac{dx}{1+x^2} = \tan^{-1} x + C$

$\frac{d}{dx}(-\cot^{-1} x) = \frac{1}{1+x^2}$; $\int \frac{dx}{1+x^2} = -\cot^{-1} x + C$

<u>TRIPLE INTEGRATION :</u>

<u>EXAM</u> :

$$\int_0^1 \int_0^x \int_{-y^2}^{x^2} (x+1)\,dz\,dy\,dx$$

$$= \int_0^1 \int_0^x (x+1)z \Big|_{-y^2}^{x^2} dydx = \int_0^1 \int_0^x (x+1)x^2 + (x+1)y^2\, dydx$$

$$= \int_0^1 \int_0^x x^3 + x^2 + xy^2 + y^2\, dydx = \int_0^1 x^3 y + x^2 y + \frac{1}{3}xy^3 + \frac{1}{3}y^3 \Big|_0^x dx$$

$$= \int_0^1 x^4 + x^3 + \frac{1}{3}x^4 + \frac{1}{3}x^3\, dx = \int_0^1 \frac{4}{3}x^4 + \frac{4}{3}x^3\, dx = \frac{4}{15}x^5 + \frac{1}{3}x^4 \Big|_0^1 = \frac{9}{15}$$

<u>EXAM</u> :

$$\int_0^1\int_0^x\int_{-y^2}^{x^2} dzdydx$$

$$=\int_0^1\int_0^x z\Big|_{-y^2}^{x^2} dydx = \int_0^1\int_0^x x^2 + y^2\, dydx$$

$$=\int_0^1 x^2 y + \frac{1}{3}y^3\Big|_0^x dx = \int_0^1 x^3 + \frac{1}{3}x^3\, dx$$

$$=\int_0^1 \frac{4}{3}x^3\, dx = \frac{1}{3}x^4\Big|_0^1 = \frac{1}{3}$$

$$=\int_0^1 x^4 + x^3 + \frac{1}{3}x^4 + \frac{1}{3}x^3\, dx = \int_0^1 \frac{4}{3}x^4 + \frac{4}{3}x^3\, dx = \frac{4}{15}x^5 + \frac{1}{3}x^4\Big|_0^1 = \frac{9}{15}$$

Home works

1) Find the equation of the curve whose slope at any point p(x,y) is
$m = (x^4 + 16x + 4)^2 (x^3 + 4)$
and passing through the point (2,1) .

2) Find the equation of the curve whose slope at any point p(x,y) is
$m = x(x+5)^2$
and passing through the point (2,1) .

3) Find :

$$\int_0^1\int_0^3 x\sqrt{x^2 + y}\, dydx$$

$$\int_0^1\int_{-1}^{\sqrt{y}} y\, dxdy$$

$$\int_0^1\int_x^{x^2}\int_{x-y}^{x+y} (x + 2y + 4z)\, dzdydx$$

$$\int_0^1\int_0^2\int_0^3 (z^3 y^2 x)\, dxdydz$$

$$\int \frac{1}{x}\,dx = Ln\left|x\right| + C$$

and in general form

$$\int \frac{1}{u}\frac{du}{dx} = Ln\left|u\right| + C$$

EXAM : Find

$$\int \frac{dx}{2x+1} = \frac{1}{2}\int \frac{2dx}{2x+1} = \frac{1}{2}Ln(2x+1) + C$$

$$\int \frac{x^3+4x+1}{x}\,dx$$

عندما تكون درجة البسط اعلى من درجة المقام نقسم اولاً ثم نكامل .

$$= \int \frac{x^3}{x}\,dx + \int \frac{4x}{x}\,dx + \int \frac{1}{x}\,dx$$

$$= \int x^2\,dx + \int 4\,dx + \int \frac{1}{x}\,dx = \frac{x^3}{3} + 4x + Ln(x) + C$$

$$\int \frac{x}{(x^2+1)^3}\,dx = \int x(x^2+1)^{-3}dx = \frac{1}{2}\int 2x(x^2+1)^{-3}dx = \frac{1}{2}\frac{(x^2+1)^{-2}}{-2} + C$$

Home work

$$Find \int \frac{x^2+2x+4}{x^3+3x^2+12x+10}\,dx \quad , \int \frac{8x+18}{(2x-1)(x+5)}\,dx$$

تكاملات الدوال الاسية :

$$\int e^x\,dx = e^x + C$$

and in general form :

$$\int e^u\,du = e^u + C$$

EXAM : Find

$$\int e^{4x}dx = \frac{1}{4}\int 4e^{4x}dx = \frac{1}{4}e^{4x} + C$$

$$\int e^{-2x}dx = \frac{-1}{2}\int -2e^{-2x}dx = \frac{-1}{2}e^{-2x} + C$$

$$\int e^{2Lnx}dx = \int e^{Lnx^2}dx = \int x^2 dx = \frac{x^3}{3} + C$$

$$\int x^2 e^{-2x^3}dx = \frac{-1}{6}\int -6x^2 e^{-2x^3}dx = \frac{-1}{6}e^{-2x^3} + C$$

$$\int \frac{e^x + e^{-x}}{e^x - e^{-x}}dx = Ln\left|e^x - e^{-x}\right| + C$$

تكامل الدالة الاسية من نوع a^x

$$\int a^u du = \frac{a^u}{Lna} + C \quad , \; a > 0 \; , \; a \neq 1$$

EXAM : Find

$$\int 3^x dx = \frac{3^x}{Ln3} + C$$

$$\int 7^{x^2} x\,dx = \frac{1}{2}\int 7^{x^2} 2x\,dx = \frac{1}{2}\frac{7^{x^2}}{Ln7} + C$$

EXAM

$$\int \frac{dx}{1+16x^2} = \frac{1}{4}\int \frac{4dx}{1+(4x)^2} = \frac{1}{4}\tan^{-1}(4x)+C$$

$$\int \frac{dx}{9+4x^2} = \frac{1}{2}\cdot\frac{1}{3}\tan^{-1}\frac{2x}{3}+C$$

$$\int \frac{e^x dx}{1+e^{2x}} = \int \frac{e^x dx}{1+(e^x)^2} = \tan^{-1}(e^x)+C$$

$$\int \frac{dx}{\sqrt{e^{2x}-1}} = \int \frac{e^x dx}{e^x\sqrt{e^{2x}-1}} = \sec^{-1}\left|e^x\right|+C$$

$$\int \frac{dx}{\sqrt{x}\sqrt{4-x}} = 2\int \frac{dx}{2\sqrt{x}\sqrt{4-x}} = 2\cdot\frac{1}{2}\sin^{-1}\frac{\sqrt{x}}{2} = \sin^{-1}\frac{\sqrt{x}}{2}+C$$

$$\int \frac{x}{3+x^4}dx = \frac{1}{2}\int \frac{2x}{3+(x^2)^2}dx = \frac{1}{2}\frac{1}{\sqrt{3}}\tan^{-1}\frac{x^2}{\sqrt{3}} = \frac{1}{2\sqrt{3}}\tan^{-1}\frac{x^2}{\sqrt{3}}+C$$

Home Work

Find

$$\int \frac{\sec^2 x}{\sqrt{1-\tan^2 x}}dx \quad , \quad \int \frac{e^{-x}}{\sqrt{1-e^{-2x}}}dx \quad , \quad \int \frac{dx}{x\sqrt{1-Ln^2 x}}$$

$$\int \frac{\sin x}{\cos^2 x+1}dx \quad , \quad \int \frac{1+\tan^2 x}{\sqrt{1-\tan^2 x}}dx \quad , \quad \int \frac{\sin x\cos x}{1+\cos^2(2x)}dx$$

$$\int \frac{e^x\,dx}{1+e^{2x}} = \int \frac{e^x\,dx}{1+(e^x)^2} = \tan^{-1}(e^x)+C$$

$$\int \frac{dx}{\sqrt{e^{2x}-1}} = \int \frac{e^x\,dx}{e^x\sqrt{e^{2x}-1}} = \sec^{-1}\left|e^x\right|+C$$

$$\int \frac{dx}{\sqrt{x}\,\sqrt{4-x}} = 2\int \frac{dx}{2\sqrt{x}\,\sqrt{4-x}} = 2\cdot\frac{1}{2}\sin^{-1}\frac{\sqrt{x}}{2} = \sin^{-1}\frac{\sqrt{x}}{2}+C$$

$$\int \frac{x}{3+x^4}\,dx = \frac{1}{2}\int \frac{2x}{3+(x^2)^2}\,dx = \frac{1}{2}\frac{1}{\sqrt{3}}\tan^{-1}\frac{x^2}{\sqrt{3}} = \frac{1}{2\sqrt{3}}\tan^{-1}\frac{x^2}{\sqrt{3}}+C$$

Home Work

Find

$$\int \frac{\sec^2 x}{\sqrt{1-\tan^2 x}}dx \quad , \quad \int \frac{e^{-x}}{\sqrt{1-e^{-2x}}}dx \quad , \quad \int \frac{dx}{x\sqrt{1-Ln^2 x}}$$

$$\int \frac{\sin x}{\cos^2 x+1}dx \quad , \quad \int \frac{1+\tan^2 x}{\sqrt{1-\tan^2 x}}dx \quad , \quad \int \frac{\sin x\,\cos x}{1+\cos^2(2x)}dx$$

HYPERBOLIC TRIG. FUN. INTEGRATION :

$\langle 1\rangle$ $\int \sinh u \cdot du = \cosh u + C$

$\langle 2\rangle$ $\int \cosh u \cdot du = \sinh u + C$

$\langle 3\rangle$ $\int \text{sech}^2 u \cdot du = \tanh u + C$

$\langle 4\rangle$ $\int \text{csch}^2 u \cdot du = -\coth u + C$

$\langle 5\rangle$ $\int \text{sech} u \tanh u \cdot du = -\text{sech} u + C$

$\langle 6\rangle$ $\int \text{csch} u \coth u \cdot du = -\text{csch} u + C$