المحاضرة (7)

Reasons for discontinuity (عدم الاتصال) اسباب الانفصال

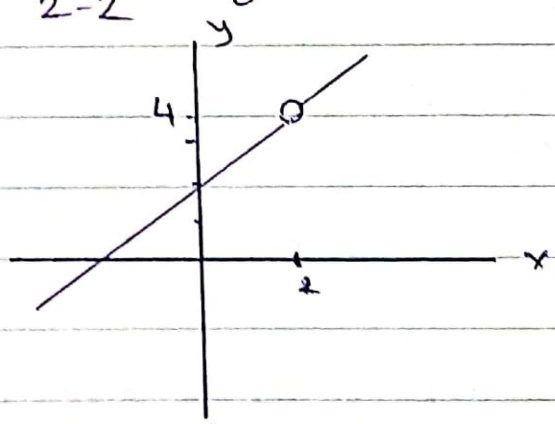
① the Limit of f(x) at (a) may fail to exist
عدم وجود النهاية

② the limit of f(x) at a may exist but f(x) may not defined at (a)

③ - the limite of f(x) at a may exist and f(a) may by defined at (a) but the Limite may not equal a

EX: $f(x) = \frac{1}{x-2}$ is continuous at 2

① $f(2) = \frac{1}{2-2} = \frac{1}{0}$ الدالة تكون غير معرفة عند $x=2$



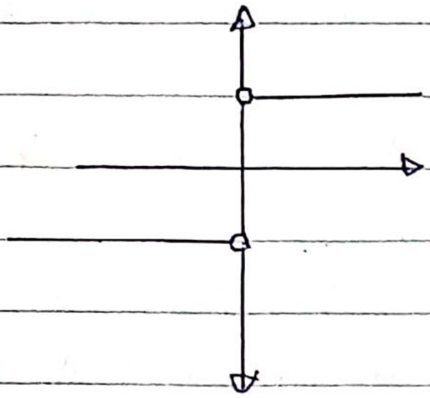
then f(x) discontinuous at $x=2$

EX: $f(x) = \frac{|x|}{x}$ at $x=0$

1- $f(0) = \frac{0}{0}$ is not defind

2- $\lim\limits_{x \to 0^+} f(x) = \lim\limits_{x \to 0^+} \frac{x}{x} = 1$, $\lim\limits_{x \to 0^-} f(x) = \lim\limits_{x \to 0^-} \frac{-x}{x} = -1$

thus $\lim\limits_{x \to 0^-} f(x) \neq \lim\limits_{x \to 0^+} f(x)$



Ex: $f(x) = \begin{cases} x^2 & \text{if } x \geq 0 \\ -x^2 & \text{if } x < 0 \end{cases}$

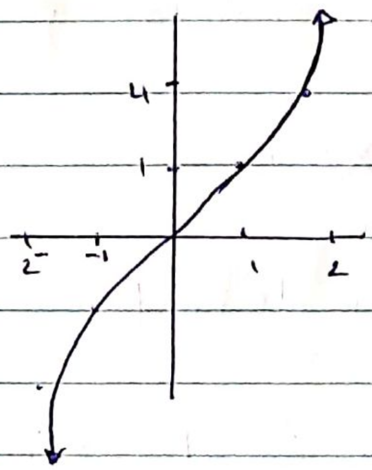
1) $f(0) = 0$

2) $\lim\limits_{x \to 0} f(x) = \lim\limits_{x \to 0^+} x^2 = 0$

$\lim\limits_{x \to 0^-} f(x)\,(x) = \lim\limits_{x \to 0^-} -x^2 = 0$

$\therefore \lim\limits_{x \to 0^-} f(x) = \lim\limits_{x \to 0^+} f(x) = f(0)$

$\therefore$ The function is continuous



Ex a) $f(x) = \begin{cases} x^2 - 1 & \text{if } x \leq 0 \\ x & \text{if } x > 0 \end{cases}$ b) $f(x) = \begin{cases} 1 & \text{if } x \geq 0 \\ -1 & \text{if } x < 0 \end{cases}$

c) $f(x) = |x|$, d) $\begin{cases} \dfrac{x^2-4}{x^2+2} & \text{if } x \neq -2 \\ 0 & \text{if } x = -2 \end{cases}$

e) $f(x) = \begin{cases} \dfrac{x^2-4}{x+2} & \text{if } x \neq 2 \\ -4 & \text{if } x = -2 \end{cases}$

Theorem in Continuty

If $f(x)$ and $g(x)$ are continuous function for a then $(f+g)(x)$ and $(f-g)$ and $(f.g)$ are continuous function at a, also $\frac{f}{g}$ is continuous at a when $g(a) \neq 0$

continuty on interval

يقال للدالة مستمرة في فترة (مفتوحة، مغلقة، نصف مفتوحة)
اذا كانت مستمرة في كل نقطة من نقاط تلك الفترة

Ex : $f(x) = [x]$ at $x = 2$

① $f(2) = 2$

② $\lim_{x \to 2} [x] = 1$ $\therefore f(2) \neq \lim_{x \to 2} [x]$ not cont.

$$\lim_{x \to 2.5^-} [x] = 2$$

$$\lim_{x \to 2.5^+} [x] = 2 \quad \text{exist}$$

$f(2.5) = [2.5] = 2$ $\quad \therefore f(2.5) = \lim_{x \to 2.5} f(x) = 2$

∴ دالة العدد الصحيح الاكبر مستمرة في $x = 2.5$

$x = 0$ $\quad f(0) = [0] = 0$

$$\lim_{x \to 0^-} f(x) = [x] = -1$$

$\lim_{x \to 0^+} f(x) = \lim_{x \to 0^+} [x] = 0$ $\quad \therefore \lim_{x \to 0^-} [x] \neq \lim_{x \to 0^+} [x]$ note not exist

لاتوجد غاية للدالة $[x]$ عند $x = 0$

وعليه تكون الدالة مستمرة في الفترات المفتوحة بين كل عددين صحيحين مثلاً في الفترة $(0,1)$

Some continuous function

$f(x) = k$

① $f(k) = k$

$\lim_{x \to k} f(x) = k$ موجودة ① = ②

$$\therefore f(k) = \lim_{x \to k} f(x)$$

∴ الداله مستمرة

(2) $f(x)=x$ at $x=a$

(1) $f(a)=a$

(2) $\lim_{x\to a} f(x)=a$ exist

$\therefore f(a)=\lim_{x\to a} f(x)$, f is continuous

Properties of continuity

If $f(x)$ and $g(x)$ are real valued continuous function then

1) $F(x) = f(x) \mp g(x)$

2) $F(x) = f(x) . g(x)$

3) $F(x) = \frac{f(x)}{g(x)}$, if $g(x) \neq 0$

are continuous function

Proof:

1) $\lim_{x\to c} F(x) = \lim_{x\to c} (f(x) \mp g(x))$

$= \lim_{x\to c} f(x) \mp \lim_{x\to c} g(x)$ exist

$= f(c) \mp g(c)$

(1) $F(c) = f(c) \mp g(c)$ (1)=(2) continuous

continuity over a closed interval

Def: A function f is continuous over [a,b] if:

1. f is continuous at each point of the open interval (a,b)

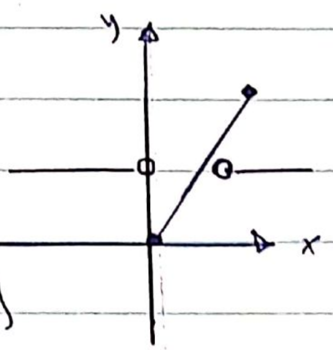
2) f is continuous on the right at (a)

3) f is continuous on the left at (b)

Example:-

The function $f(x) = \begin{cases} 2x & \text{if } 0 \le x \le 1 \\ 1 & \text{otherwise} \end{cases}$

f is not cont. at $x = 0$ and $x = 1$



$f(1) = 1$ for all x except $x = \{0, 1\}$

find the value of k that makes $f(x)$ cont at $x=3$

$$f(x) = \begin{cases} \frac{x^2-9}{x-3} & x<3 \\ kx-3 & x \geq 3 \end{cases}$$

Solution

since $f(x)$ is cont at $x=3$

$$\Rightarrow \lim_{x\to 3} f(x) = f(3)$$

$$\lim_{x\to 3} \frac{x^2-9}{x-3} = 3k-3$$

$$\lim_{x\to 3} \frac{\cancel{(x-3)}(x+3)}{\cancel{(x-3)}} = 3k-3$$

$$\lim_{x\to 3} x+3 = 3k-3$$

$$6 = 3k-3 \Rightarrow 9=3k = k=\frac{9}{3}=3$$

Theorem:

Every polynomial function

$$f(x)=a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0$$

is continuous

continuous function

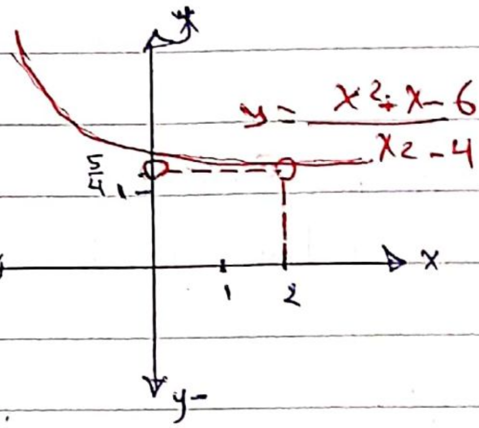
Ex: If $f(x)=\dfrac{x^2+x-6}{x^2-4}$, what value should $f(2)$ have

$$f(x)=\frac{x^2+x-6}{x^2-4}=\frac{(x-2)(x+3)}{(x-2)(x+2)}=\frac{x+3}{x+2}$$

there fore

$$\lim_{x\to 2}\frac{x+3}{x+2}=\frac{5}{4}$$

and $f(2)$ will make $\frac{5}{4}$



$$f(2)=\lim_{x\to 2}f(x)=\frac{5}{4}$$

$$f(x)=\begin{cases}\dfrac{x^2+x-6}{x^2-4} & \text{if } x\neq 2\\[2mm] \dfrac{5}{4} & \text{if } x=2\end{cases}$$

Composite of continuous function

Theorem:

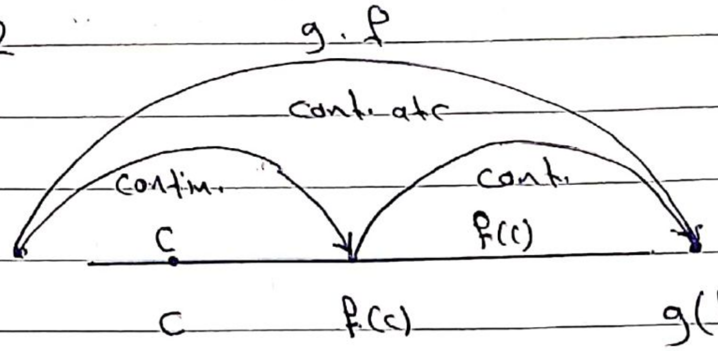
If f is continuous at c and g is continuous at $f(c)$ then the composite $g\circ f$ is continuous at c.

Example: show that the function

$y=\left|\dfrac{x\sin x}{x^2+2}\right|$ is continuous at every value of x

(8) Solution: the function y is the composite of the continuous function

$$f(x) = \frac{x + \sin x}{x^2 + 2} \quad \text{and} \quad g(x) = |x|$$



Exercises:

which the function are not continuous? 112 P: Calculor

1- $y = \dfrac{1}{x-2}$ 2) $y = \dfrac{1}{(x+2)^2}$ 3) $y = \dfrac{x}{x+1}$

4) $y = \dfrac{x+1}{x^2 - 4x + 3}$ 5) $y = \dfrac{x^3 - 1}{x^2 - 1}$ 6) $y = \dfrac{1}{x^2 + 1}$

7) $y = \dfrac{\cos x}{x}$ 8) $y = \dfrac{|x|}{x}$

Exercises:

Find the Limite.

1) $\lim\limits_{x \to 1} 5x - 4$

2) $\lim\limits_{x \to -3} |7x + 10|$

3) $\lim\limits_{x \to -2} x^2(x+1)$

4) $\lim\limits_{x \to 3} (x+2)(x-5)$

5) $\lim\limits_{x \to 2} \left(\frac{x}{x+1}\right)\left(\frac{3x+5}{x^2+x}\right)$

6) $\lim\limits_{x \to 0} \frac{\sin 2x}{4x}$

7) $\lim\limits_{x \to 0} \frac{x + \sin x}{x}$

8) $\lim\limits_{x \to \pi} \sin\left(\frac{x+\pi}{4}\right) \cos\left(\frac{x-\pi}{4}\right)$

9) $\lim\limits_{x \to \pi} \frac{x + \sin x}{x + \cos x}$

10) $\lim\limits_{x \to \infty} \frac{\sin x}{\sqrt{x}}$

11) IS the function $f(x) = \frac{|x-1|}{x-1}$ cont. at $x = 1$

$$|x-1| = \begin{cases} x-1 & x-1 \geq 0 \Rightarrow x \geq 1 \\ -(x-1) & x-1 < 0 \Rightarrow x < 1 \end{cases}$$

$$\lim_{x \to 1^-} \frac{|x-1|}{x-1} = \frac{-(x-1)}{x-1} = -1 \quad \text{exist}$$

$$\lim_{x \to 1^+} \frac{|x-1|}{x-1} = \frac{x-1}{x-1} = +1 \quad \text{exist}$$

$$\therefore \lim_{x \to 1^+} \frac{|x-1|}{x-1} \neq \lim_{x \to 1^-} \frac{|x-1|}{x-1}$$

$\therefore f$ is not continuous

Increments الزيادات

If x_1 and x_2 are two numbers then value of x that changed from x_1 to x_2, then $x_2 - x_1$ is the increment of x,

$\Delta x = x_2 - x_1$ ويشار لمقدار التغير بالرمز

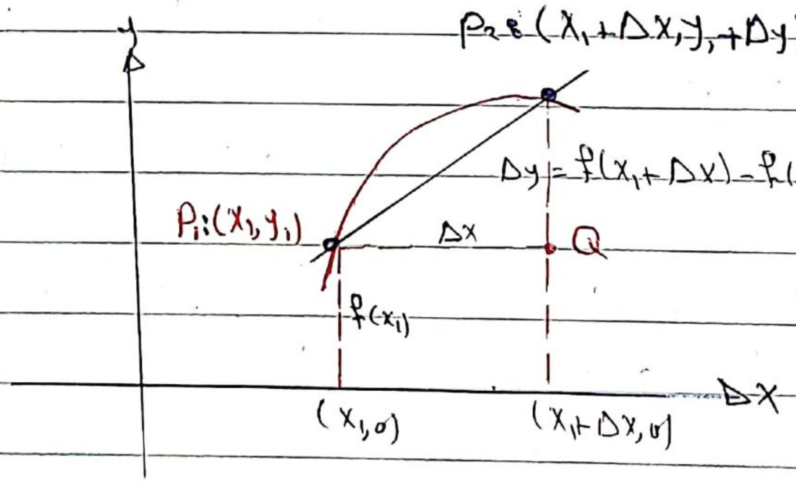
Def:- تعريف: اذا كانت $y = f(x)$ وكان x_1 , $x_1 + \Delta x$ عددين في مجال f ، فان

$$\Delta y = f(x_1 + \Delta x) - f(x_1)$$

هي الزيادة للمتغير التابع التي تقابل الزيادة Δx للمتغير المستقل x من x_1 والذي يكافئ

$$\Delta f = f(x_1 + \Delta x) - f(x_1)$$

ميل القاطع $= \dfrac{\Delta y}{\Delta x}$



$$m = \frac{\Delta y}{\Delta x} = \frac{f(x_1 + \Delta x) - f(x_1)}{\Delta x}$$

تعريف القاطع: هو المستقيم الذي يقطع المنحني بنقطتين او اكثر.

تعريف المماس: هو المستقيم الذي يمس المنحني بنقطة واحدة او اكثر

Increments الزيادات

$y = f(x)$

Δx: the increment of x

Δy: the increment of y

$\cancel{y} + \Delta y = f(x + \Delta x)$

$\mp \cancel{y} = \mp f(x)$ بالطرح

$\Delta y = f(x + \Delta x) - f(x)$

Ex: If $f(x) = x^2$ find Δy if x increament from 3 to 3.1

Sol 1

$\Delta x = 3.1 - 3 = 0.1$

$x = 3$

$y + \Delta y = f(x + \Delta x)$

$= (x + \Delta x)^2$

$\cancel{y} + \Delta y = \cancel{x^2} + 2x\Delta x + \Delta x^2$

$\cancel{y} = -\cancel{x^2}$ بالطرح

$\therefore \Delta y = 2x \cdot \Delta x + (\Delta x)^2$

$\Delta y = 2(3) \cdot (0.1) + (0.1)^2$

$= 6 \cdot (0.1) + 0.01$

$= 0.61$