المحاضرة (8)

Differentiation الاشتقاق

Def. The derivative of a function f is the function $\bar{f}$ whose value at x is defined by the equation

$$\bar{f}(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

the function $\frac{f(x+h) - f(x)}{h}$ is the difference quotient for f at x

differentiable at a point

a function that has a derivative at a point x is said to be differentiable at x

اذا كانت الدالة غير متصلة عند نقطة x تسمى قابلة للاشتقاق عند x

differentiable function

A function that is differentiable at every point of its domain is called differentiable

اذا كانت الدالة غير متصلة عند كل نقطة في مجال الدالة

the slop and tangent. تسمى قابلة للاشتقاق

When the number $\bar{f}(x)$ exist, it is called the slop of the curve $y = f(x)$ at x, the line through the point $(x, f(x))$ with slope $\bar{f}(x)$, is the tangent to the curve at x.

E اذا كان $\bar{f}(x)$ موجود يسمى المماس الذي ميله $\bar{f}(x)$ عند x هو المستقيم الذي يمر بالنقطة $(x, f(x))$ وميله $\bar{f}(x)$ هو ظل الزاوية التي يصنعها المماس عند النقطة x

Ex: Find $\frac{dy}{dx}$ if $y = 1/x$

Solution

$$f(x) = \frac{1}{x}, \quad f(x+\Delta x) = \frac{1}{x+\Delta x}$$

$$f'(x) = \frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{h}$$

$$= \lim_{\Delta x \to 0} \frac{\frac{1}{x+\Delta x} - \frac{1}{x}}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{x - x - \Delta x}{x(x+\Delta x)} \cdot \frac{1}{\Delta x} = \lim_{\Delta x \to 0} \frac{-\Delta x}{\Delta x (x^2 + \Delta x)}$$

$$= \frac{-1}{x^2}$$

Ex: find $f'(x)$ if $f(x) = x^2 - 2x$

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$$f(x+\Delta x) = (x+\Delta x)^2 - 2(x+\Delta x)$$

$$= x^2 + 2x\Delta x + \Delta x^2 - 2x - 2\Delta x$$

$$f'(x) = \lim_{\Delta x \to 0} \frac{x^2 + 2x\Delta x + \Delta x^2 - 2x - 2\Delta x - x^2 + 2x}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\Delta x^2 + 2x\Delta x - 2\Delta x}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\Delta x(\Delta x + 2x - 2)}{\Delta x}$$

$$f'(x) = 2x - 2$$

H.W: Show that the derivative of $y = \frac{1}{\sqrt{x}}$ is

$$\frac{dy}{dx} = \frac{1}{2\sqrt{x}}$$

Theorem:-

If f has a derivative at $x=c$, then f is continuous at $x=c$

Proof: with out proof

Exercises: p:140

1) $y = 3x^2$

2) $y = \frac{x}{x+1}$

3) $x + \frac{9}{x}$

4) $y = \sqrt{2x}$

5) $y = \frac{1}{\sqrt{2x+3}}$

6) $y = \frac{x^2}{2}$

7) $y = \frac{1}{2x+1}$

8) $y = x - \frac{1}{x}$

9) $y = \sqrt{x+1}$

10) $y = \sqrt{2x+3}$

Differentiation Rules قواعد الاشتقاق

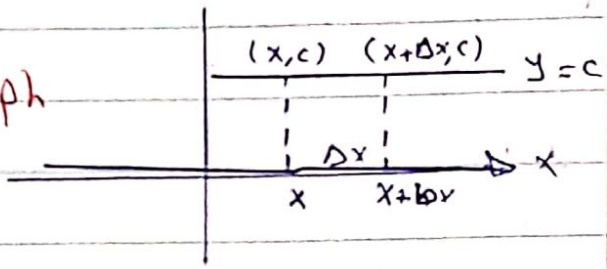
1- If c is a constant, then $\frac{d}{dx}(c) = 0$

Proof:-

$f(x) = c$

$$\Rightarrow f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x} = \frac{c-c}{\Delta x} = 0$$

Note: the slope of the graph of $y = c$ is Zero at every point.



2- If n is a positive integer, then

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

1- $\frac{d}{dx}(x) = \frac{d}{dx}(x^1) = 1x^0 = 1$

2) $\frac{d}{dx}(x^2) = 2x^1 = 2x$

3) $\frac{d}{dx}(x^3) = 3x^2$

4) $\frac{d}{dx}(x^4) = 4x^3$

5) $\frac{d}{dx}(x^5) = 5x^4$

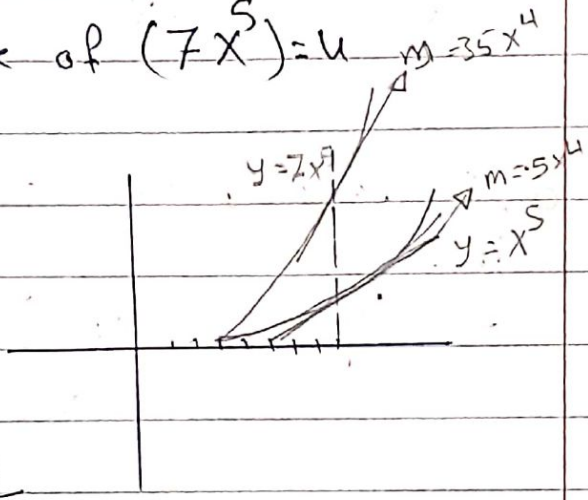
3) If u is differentiable function of x and c is a constant, then

$\frac{d}{dx}(cu) = c\frac{du}{dx}$, if n is a positive integer then

$\frac{d}{dx}(cx^n) = cnx^{n-1}$

Example: Find the derivative of $(7x^5) = u$

$\frac{d}{dx}(7x^5) = 35x^4$



2) $\frac{d}{dx}(-u) = \frac{d}{dx}(-1 \cdot u)$

$= (-1)\frac{du}{dx} = -\frac{du}{dx}$

EX:1 $y = x^4 + 12x$

بعد الجمع نأخذ المشتقة

$\frac{dy}{dx} = 4x^3 + 12$ بعد القاعدة (4)

EX 2: $y = \frac{7x^2}{3} - 5$

$\frac{dy}{dx} = m = \frac{14x}{3}$

EX3: $y = x^3 + 3x^2 - 5x + 1$

$\frac{dy}{dx} = 3x^2 + 6x - 5$

4) If u and v are differentiable function of x then their sum and difference are differentiable at every point where u and v are both differentiable

1) $\frac{d}{dx}(u+v) = \frac{du}{dx} + \frac{dv}{dx}$

2) $\frac{d}{dx}(u-v) = \frac{du}{dx} - \frac{dv}{dx}$

6) If u and v are differentiable function of x then

مشتقة حاصل قسمة دالتين

$$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\left(\frac{du}{dx}\right) - u\frac{dv}{dx}}{v^2} \qquad v \neq 0$$

مشتقة حاصل ضرب دالتين

5) $= \frac{d}{dx}(u.v) = u.\frac{dv}{dx} + v\frac{du}{dx}$

7) $\frac{d}{dx}(f \circ g) = \bar{f}(g(x)).\bar{g}(x)$

$$(f \circ g)(x) = f(g(x)) \Rightarrow \frac{dy}{dx} \Rightarrow \bar{f}(g(x)).\bar{g}(x)$$

Ex: $y = \frac{3x+5}{x^2+4}$, find $\frac{dy}{dx}$

$$\frac{dy}{dx} = \frac{(x^2+4)3 - (3x+5)(2x)}{(x^2+4)^2}$$

$$\frac{dy}{dx} = \frac{3x^2+12-6x^2-10x}{(x^2+4)^2} = \frac{12-10x-3x^2}{(x^2+4)^2}$$