$$\int \frac{e^x\,dx}{1+e^{2x}} = \int \frac{e^x\,dx}{1+(e^x)^2} = \tan^{-1}(e^x) + C$$

$$\int \frac{dx}{\sqrt{e^{2x}-1}} = \int \frac{e^x\,dx}{e^x\sqrt{e^{2x}-1}} = \sec^{-1}\left|e^x\right| + C$$

$$\int \frac{dx}{\sqrt{x}\,\sqrt{4-x}} = 2\int \frac{dx}{2\sqrt{x}\,\sqrt{4-x}} = 2\cdot\frac{1}{2}\sin^{-1}\frac{\sqrt{x}}{2} = \sin^{-1}\frac{\sqrt{x}}{2} + C$$

$$\int \frac{x}{3+x^4}\,dx = \frac{1}{2}\int \frac{2x}{3+(x^2)^2}\,dx = \frac{1}{2}\frac{1}{\sqrt{3}}\tan^{-1}\frac{x^2}{\sqrt{3}} = \frac{1}{2\sqrt{3}}\tan^{-1}\frac{x^2}{\sqrt{3}} + C$$

Home Work

Find

$$\int \frac{\sec^2 x}{\sqrt{1-\tan^2 x}}dx \quad , \quad \int \frac{e^{-x}}{\sqrt{1-e^{-2x}}}dx \quad , \quad \int \frac{dx}{x\sqrt{1-Ln^2 x}}$$

$$\int \frac{\sin x}{\cos^2 x+1}dx \quad , \quad \int \frac{1+\tan^2 x}{\sqrt{1-\tan^2 x}}dx \quad , \quad \int \frac{\sin x\,\cos x}{1+\cos^2(2x)}dx$$

HYPERBOLIC TRIG. FUN. INTEGRATION :

$\langle 1\rangle \int \sinh u \cdot du = \cosh u + C$

$\langle 2\rangle \int \cosh u \cdot du = \sinh u + C$

$\langle 3\rangle \int \operatorname{sech}^2 u \cdot du = \tanh u + C$

$\langle 4\rangle \int \operatorname{csch}^2 u \cdot du = -\coth u + C$

$\langle 5\rangle \int \operatorname{sech} u \tanh u \cdot du = -\operatorname{sech} u + C$

$\langle 6\rangle \int \operatorname{csch} u \coth u \cdot du = -\operatorname{csch} u + C$

$\lfloor 1 \rfloor \int \cosh 7x\,dx = \frac{1}{7}\sinh 7x + c$

$\lfloor 2 \rfloor \int \tanh x \ \operatorname{sech}^2 x\,dx = \frac{1}{2}\tanh^2 x + c$

$\lfloor 3 \rfloor \int \frac{\sinh(Lnx)}{x}dx = \cosh(Lnx) + c$

INVERSE HYPERBOLIC TRIG. FUN. INTEGRATION :

$$\langle 1 \rangle \int \frac{du}{\sqrt{a^2+u^2}} = \sinh^{-1}(\frac{u}{a}) + C$$

$$\langle 2 \rangle \int \frac{du}{\sqrt{u^2-a^2}} = \cosh^{-1}(\frac{u}{a}) + C$$

$$\langle 3,4 \rangle \int \frac{du}{a^2-u^2} = \begin{cases} \frac{1}{a}\tanh^{-1}(\frac{u}{a}) \\ \frac{1}{a}\coth^{-1}(\frac{u}{a}) \end{cases} + C$$

$$\langle 5 \rangle \int \frac{du}{u\sqrt{a^2-u^2}} = -\frac{1}{a}\operatorname{sech}^{-1}\left|\frac{u}{a}\right| + C$$

$$\langle 6 \rangle \int \frac{du}{|u|\sqrt{a^2+u^2}} = -\frac{1}{a}cs\,\mathrm{ch}^{-1}\frac{u}{a} + C$$

Examples

1) $\int \frac{dx}{\sqrt{1+4x^2}} = \frac{1}{2}\sinh^{-1}(2x) + C$

2) $\int \frac{dx}{\sqrt{9+16x^2}} = \frac{1}{4}\sinh^{-1}(\frac{4x}{3}) + C$

3) $\int \frac{e^x}{1-e^{2x}}dx = \tanh^{-1}(e^x) + C$

4) $\int \frac{dx}{9-x^2} = \frac{1}{3}\tanh^{-1}\frac{x}{3} + C$

HYPERBOLIC TRIG. FUN. INTEGRATION :

$\langle 1 \rangle \int \sinh u \cdot du = \cosh u + C$

$\langle 2 \rangle \int \cosh u \cdot du = \sinh u + C$

$\langle 3 \rangle \int \text{sech}^2 u \cdot du = \tanh u + C$

$\langle 4 \rangle \int \text{csch}^2 u \cdot du = -\coth u + C$

$\langle 5 \rangle \int \sec hu \tanh u \cdot du = -\text{sech} u + C$

$\langle 6 \rangle \int \csc hu \coth u \cdot du = -\text{csch} u + C$

Examples

$\lfloor 1 \rfloor \int \cosh 7x dx = \frac{1}{7} \sinh 7x + c$

$\lfloor 2 \rfloor \int \tanh x \sec h^2 x dx = \frac{1}{2} \tanh^2 x + c$

$\lfloor 3 \rfloor \int \frac{\sinh(Lnx)}{x} dx = \cosh(Lnx) + c$

INTEGRATION METHODS

1) Integration by part

تستخدم هذه الطريقة في حالة عدم استطاعتنا تكامل الدوال بالقواعد السابقة وملخص الطريقة اننا نجزء الدالة المعطاة الى جزأين بحيث يكون الجزء الاول قابل للاشتقاق u والجزء الثاني قابل للتكامل dv من ثم نطبق القاعدة التالية :

EXAM :

$\boxed{1} \int x e^x dx$

الان سوف نجزء هذه الدالة الى جزأين الاول u قابل للاشتقاق وهو x والثاني dv قابل للتكامل وهو $e^x dx$

$$u = x \qquad dv = e^x dx$$

$$du = dx \qquad v = e^x$$

$$\int x e^x dx = x e^x - \int e^x dx$$

$$= x e^x - e^x + C$$

$\boxed{2}\ \int x^2 e^x\,dx$

$u = x^2 \qquad dv = e^x\,dx$

$du = 2x\,dx \qquad v = e^x$

$$\int x^2 e^x\,dx = x^2 e^x - 2\int x e^x\,dx$$

$u = x \qquad dv = e^x\,dx$

$du = dx \qquad v = e^x$

$$\int x^2 e^x\,dx = x^2 e^x - 2\left(x e^x - e^x\right) + C$$

$\boxed{3}\ \int Lnx\,dx$

$u = Lnx \qquad dv = dx$

$du = \frac{1}{x}dx \qquad v = x$

$$\int Lnx\,dx = xLnx - \int dx = xLnx - x + C$$

$\boxed{4}\ \int x \sin x\,dx$

$u = x \qquad dv = \sin x\,dx$

$du = dx \qquad v = -\cos x$

$$\int x \sin x dx = -x \cos x + \int \cos x dx = -x \cos x + \sin x + C$$

$\boxed{5}\ \int x \cos x\,dx$

$u = x \qquad dv = \cos x\,dx$

$du = dx \qquad v = \sin x$

$$\int x \cos x dx = x \sin x - \int \sin x dx = x \sin x + \cos x + C$$

$\boxed{6}$ $\int e^x \cos x\, dx$

$$u = e^x \qquad dv = \cos x\, dx$$

$$du = e^x dx \qquad v = \sin x$$

$$\int e^x \cos x dx = e^x \sin x - \int e^x \sin x dx \quad \cdots\cdots\cdots(1)$$

$$u = e^x \qquad dv = \sin x\, dx$$

$$du = e^x dx \qquad v = -\cos x$$

$$\int e^x \sin x dx = -e^x \cos x + \int e^x \cos x dx \quad \cdots\cdots\cdots(2)$$

$$\int e^x \cos x dx = e^x \sin x + e^x \cos x - \int e^x \cos x dx \qquad [sub.\ (2) in (1)]$$

$$2\int e^x \cos x dx = e^x \sin x + e^x \cos x$$

$$\therefore \int e^x \cos x dx = \frac{e^x}{2}(\sin x + \cos x) + C$$

$\boxed{7}$ $\int x^2 \sin x\, dx$

$$u = x^2 \qquad dv = \sin x dx$$

$$du = 2x dx \qquad v = -\cos x$$

$$\int x^2 \sin x dx = -x^2 \cos x + 2\int x \cos x dx \quad \cdots\cdots\cdots(1)$$

$$u = x \qquad dv = \cos x dx$$

$$du = dx \qquad v = \sin x$$

$$\int x \cos x dx = x \sin x - \int \sin x dx = x \cos x + \cos x \quad \cdots\cdots(2)$$

$$\therefore \int x^2 \sin x dx = -x^2 \cos x + 2[x \sin x + \cos x] + C$$

$\boxed{8}$ $\int xLnx\ dx$

$$u = Lnx \qquad dv = xdx$$
$$du = \frac{1}{x}dx \qquad v = \frac{x^2}{2}$$

$$\int xLnx dx = \frac{1}{2}x^2 Lnx - \frac{1}{2}\int x dx$$
$$\int xLnx dx = \frac{1}{2}x^2 Lnx - \frac{1}{4}x^2 + C$$

$\boxed{9}$ $\int \sin^2 x\ dx$

$$u = \sin x \qquad dv = \sin x dx$$
$$du = \cos x dx \qquad v = -\cos x$$

$$\int \sin^2 x dx = -\sin x \cos x + \int \cos^2 x dx = -\sin x \cos x + \int (1 - \sin^2 x) dx$$
$$= -\sin x \cos x + \int dx - \int \sin^2 x dx$$
$$2\int \sin^2 x dx = -\sin x \cos x + x \Rightarrow \int \sin^2 x dx = \frac{1}{2}(x - \sin x \cos x) + C$$

$\boxed{10}$ $\int \sin^{-1} x\ dx$

$$u = \sin^{-1} x \qquad dv = dx$$
$$du = \frac{1}{\sqrt{1-x^2}}dx \qquad v = x$$

$$\int \sin^{-1} x\ dx = x \sin^{-1} x - \int \frac{x}{\sqrt{1-x^2}}dx = x \sin^{-1} x + \sqrt{1-x^2} + C$$

Home Work

$\int \cos^2 x dx$ $\int \sin^3 x dx$

$\int Ln^2 x dx$ $\int x^3 \sin x dx$

$\int x^3 e^x dx$ $\int x^2 e^{-x} dx$

$\int \tan^{-1} x dx$ $\int x \sec^{-1} x dx$

$\int \cos(Lnx) dx$ $\int (x^2 + x + 1)\sin x dx$

INTEGRAL METHODS\PARATIAL FRACTIONS

اذا كانت الدالة نسبية أي بشكل $h(x) = \frac{f(x)}{g(x)}$ **وكانت درجة البسط اقل من درجة المقام ، والمقام يتحلل الى العوامل الاولية ولم نستطع تكاملها بالطرق الاعتيادية البسيطة نلجأ الى طريقة تجزئة الكسور .**

ملاحظة : اذا كانت درجة البسط اكبر من درجة المقام او مساوية لها فنقسم اولاً ثم نكامل .

<u>الحالة الاولى</u>

اذا كانت عوامل g(x) خطية اولية (من الدرجة الاولى) وغير متكررة أي ان :

$$h(x) = \frac{f(x)}{g(x)} = \frac{A}{(x - a_1)} + \frac{B}{(x - a_2)} + \ldots\ldots + \frac{Z}{(x - a_n)}$$

ومن هذه الخطوة نوحد المقامات ونساوي معاملات x^n **في البسط للطرفين ونوجد قيم الثوابت A,B,…,Z ثم نكامل.**

<u>EXAM</u> :

$\boxed{1}$ $\int \frac{2x}{2x^2 + x - 6} dx$

$$\frac{2x}{2x^2 + x - 6} = \frac{A}{2x - 3} + \frac{B}{x + 2} = \frac{A(x + 2) + B(2x - 3)}{(2x - 3)(x + 2)}$$

$$2x = A(x + 2) + B(2x - 3)$$

$$2 = A + 2B$$

$$0 = 2A - 3B \Rightarrow A = \frac{3}{2}B$$

$$2 = \frac{3}{2}B + 2B \Rightarrow B = \frac{4}{7} \Rightarrow A = \frac{6}{7}$$

$$\therefore \int \frac{2x}{2x^2 + x - 6} dx = \int \left(\frac{6/7}{2x - 3} + \frac{4/7}{x + 2}\right) dx$$

$$= \frac{6}{7}\frac{1}{2} Ln|2x - 3| + \frac{4}{7} Ln|x + 2| + C$$