المحاضرة (١١)

Second and Higher order Derivative.

the derivative

$y' = \frac{dy}{dx}$ is the first derivative of y with respect to x

$y'' = \frac{dy'}{dx} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d^2y}{dx^2}$: Second derivative of y with respect to x

$y''' = \frac{dy''}{dx} = \frac{d^3y}{dx^3}$: third derivative of y with respect to x

Example:- $y = x^3 - 3x^2 + 2$

first derivative : $y' = 3x^2 - 6x$

second " : $y'' = 6x - 6$

third " : $y''' = 6$

fourth " : $y^{(4)} = 0$

هذه الدالة تملك مشتقات لجميع الرتب ولكن المشتقة الخامسة والمشتقات التالية لها صفراً جميعها تساوي صفر

Ex: Do not use Quotient Rule to find the derivative of

$$y = \frac{(x-1)(x^2-2x)}{x^4}$$

solution : expand the numerator and divide by x^4

$$y = \frac{x^3 - 3x^2 + 2x}{x^4} = x^{-1} - 3x^{-2} + 2x^{-3}$$

$$\frac{dy}{dx} = y' = m = -x^{-2} + 6x^{-3} - 6x^{-4}$$

$$\frac{dy}{dx} = \frac{-1}{x^2} + \frac{6}{x^3} - \frac{6}{x^4}$$

Exercises $\qquad$ P :150

find y' and y'' of the function

1) $y = x$

2) $y = +3 - x^2$

3) $y = \frac{x^3}{3} + \frac{x^2}{2} + x$

4) $y = 2x^4 - 4x^2 - 8$

5) find all non Zero derivative of the function

جد جميع المشتقات غير الصفرية للدوال الآتية

1) $y = x^2 - x$, 2) $y = \frac{x^5}{120}$

3) $y = \frac{x^4}{2} - \frac{3}{2}x^2 - x$

Implicit differentiation

The process of finding the derivative of a dependent variable in an implicit function by differentiating each term separately

هي عملية ايجاد مشتقة متغير يعتمد في دالة ضمنية عن طريق اشتقاق كل حد على حدا (منفصل)

Ex:

العلاقة ضمنية اذا لم يكن باستطاعتنا فصل x عن y

$[y(x)]^5$

١- x مضروبة بـ y

٢- y مرفوعة لقوى غير الواحد

٣- y بزاوية

$$\frac{d}{dx}[y(x)]^5 = 5(y(x))^4 . \bar{y}(x)$$

$$x^3 + y^3 + 2xy + 1 = 0$$

$$\frac{d}{dx}(x^3) + \frac{d}{dx}(y^3) + \frac{d}{dx}(2xy) + \frac{d}{dx}(1) = 0$$

$$3x^2 + 3y^2\frac{dy}{dx} + \left(2\frac{dy}{dx}(1) + 2y(1)\right) + 0 = 0$$

$$\frac{dy}{dx}(3y^2 + 2x) = -3x^2 - 2y$$

$$\therefore \frac{dy}{dx} = \frac{-3x^2 - 2y}{3y^2 + 2x}$$

Ex: $y = f(x)$ الحالة الاعتيادية

$y = 3x^2 + 4$ (Explicit) المتغير المعتمد بدلالة المستقل

$$y + xy = x \Rightarrow y(1+x) = x \Rightarrow y = \frac{x}{x+1}$$

لا حل وحيد

اما في حالة وجود اكثر من طريقة للتعبير عن الدالة نشتقها اشتقاقا ضمنيا

$x^2 + y^2 = 4$

$y^2 = 4 - x^2 \Rightarrow y = \mp\sqrt{4 - x^2}$ هناك حلين للدالة

Such that there is more that one function of x

Ex:

$2x^3 + 3y^3 = 9xy$, find the derivative.

$6x^2 + 9y^2 y' = 9(xy' + y)$

$9y^2 y' - 9xy' = 9y - 6x^2$

$y'(9y^2 - 9x) = 9y - 6x^2$

$$y' = \frac{9y^2 - 9x}{9y - 6x^2}$$

Ex: $x^3 + y^3 = 5$ Find the derivative

$\frac{d}{dx}[x^3 + y^3] = \frac{d}{dx}(5) \quad = \frac{d}{dx}[x^3] + \frac{d}{dx}[y^3] = 0$

$3x^2 + 3y^2 \cdot \frac{dy}{dx} = 0$

$\frac{d}{dx}(3x^2 - 1)^4$

$4(3x^2 - 1)^3 \frac{d}{dx}(3x^2 - 1)$

$$\therefore \frac{dy}{dx} = \frac{-3x^2}{3y^2}$$

Chain Rule
طاقة السـلـة

In Calculus: the Chain rule is the formula for computing the derivative of the composition of two or more function, i.e

If f and g are a function, then the chain Rule expresses the derivative of the composite function $f \circ g$.

If g is differentiable at x and f is differentiable at g(x), then the composit function

$F = f \circ g$ defined by $F(x) = f(g(x))$, is

$$F'(x) = f'(g(x)) \cdot g'(x)$$

Now, If $y = f(u)$, and $u = g(x)$ are both differentiable function, then

$$\frac{dy}{dx} = \frac{dy}{du} \frac{du}{dx} \quad \text{(Leibniz notation)}$$

EX: Differentiate $y = \sin(x^2)$

case 1:

$$\frac{dy}{dx} = 2x\cos(x^2)$$

Cas2: let $y = \sin u$, $u = x^2$, thenn by chain Rule

$\therefore \frac{dy}{dx} = \sin u \cdot 2x$, $\frac{dy}{dx} 2x \sin(x^2)$

power function

If $y = [g(x)]^n$, then we can write $y = f(u)$ and $u = g(x)$, then by using Chain Rule and then the power rule

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} = n u^{n-1} \cdot \frac{du}{dx} = n[g(x)]^{n-1} \cdot g'(x)$$

Ex: Differentiate $y = (x^3 - 1)^{100}$

taking $u = x^3 - 1 = g(x)$, $n = 100$

$$\therefore \frac{dy}{dx} = \frac{d}{dx}[x^3 - 1]^{100}$$

$$= 100[x^3 - 1]^{99} \cdot \frac{d}{dx}[x^3 - 1]$$

$$= 300x^2[x^3 - 1]^{99}$$

Ex: Find the derivative of the function

$$g(t) = \left(\frac{(t-2)}{2t+1}\right)^9$$

$$g'(t) = 9\left(\frac{t-2}{2t+1}\right)^8 \cdot \frac{d}{dt}\left(\frac{t-2}{2t+1}\right)$$

$$= 9\left(\frac{t-2}{2t+1}\right)^8 \cdot \frac{(2t+1)\cdot 1 - 2(t-2)}{(2t+1)^2} = \frac{\cancel{2t}+1 \cancel{-2t}+4}{(2t+1)^2}$$

$$g'(t) = 9\left(\frac{t-2}{2t+1}\right)^8 \cdot \frac{5}{(2t+1)^2}$$

$$g'(t) = \frac{45(t-2)^8}{(2t+1)^{10}}$$

H.w : Differentiate ① $y = (2x+1)^5(x^3 - x + 1)^4$

② $y = \sin(\cos(\tan x))$

③ $y = \sqrt{\sec x^3}$

H.w:

Find the slop of the tangent to the curve

1) $x^2 - 2y^2 = 2$ at $(2,1)$

Solution:

the slop $= \frac{dy}{dx}$ at the point $(2,1)$

نشتق طرفي المعادلة بالنسبة لـ x

$$2x - 4y\frac{dy}{dx} = 0$$

$$2(2) - 4(1)\frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = \frac{-4}{-4} = 1$$

2) Given that $\cos(xy) = y - 1$, find y' at the point $(\frac{\pi}{2}, 1)$

Parametric equation,

If $y = g(t)$, $x = f(t)$, then $\dfrac{dy}{dx} = \dfrac{\frac{dy}{dt}}{\frac{dx}{dt}}$

تكون المعادلة بارامترية اذا كانت كل من f و g هي دوال بدلالة t.

Ex: If $y = 2\cos t$, $x = 3\sin t$. find $\dfrac{dy}{dx}$

Solution

$\dfrac{dy}{dt} = -2\sin t$, $\dfrac{dx}{dt} = 3\cos t$

$$\therefore \frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{-2\sin t}{3\cos t} = \frac{-2}{3}\tan t$$

Second derivative in Parametric

$$\frac{d^2y}{dx^2} = \frac{\frac{dy'}{dt}}{\frac{dx}{dt}}$$

الصيغة العامة

general formula

Ex: find $\dfrac{d^2y}{dx^2}$, if $x = t - t^2$ and $y = t - t^3$

Solution:

$$y' = \frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{1-3t^2}{1-2t}$$

$$\frac{d^2y}{dx^2} = \frac{dy'}{dx} = \frac{dy'/dt}{dx/dt} = \frac{\frac{d}{dt}\left(\frac{1-3t^2}{1-2t}\right)}{1-2t}$$

$$= \frac{(1-2t)(-6t) - (1-3t^2)(-2)}{(1-2t)} \cdot \frac{1}{1-2t}$$