Lecture - 9 -

EX: Prove

$$(1+x)^n = \sum_{r=0}^{n} C_r^n x^r$$

$$(1+x)^n = C_0^n 1^{n-0} x^0 + C_1^n 1^{n-1} x^1 + C_2^n 1^{n-2} x^2 + \dots + C_n^n 1^0 x^n$$

$$= C_0^n + C_1^n x + \dots + C_n^n x^n$$

$$(1+x)^n = \sum_{r=0}^{n} C_r^n x^r$$

EX:

$$2^n = \sum_{r=0}^{n} C_r^n$$

$$(1+1)^n = C_0^n 1^{n-0} 1^0 + C_1^n 1^{n-1} 1^1 + C_2^n 1^{n-2} 1^2 + \dots + C_n^n 1^0 1^n$$

$$= \sum_{r=0}^{n} C_r^n$$

EX:(1) Find the term in the expantion of $(3xy^2 - z^2)^7$ which contains y^6. أوجد الحد الذي يحتوي y^6 في مفكوك $(3xy^2 - z^2)^7$

$$(3xy^2 - z^2)^7 = \sum_{r=0}^{7} C_r^7 (3xy^2)^{7-r} (-z^2)^r$$

$$= C_0^7 (3xy^2)^7 (-z^2)^0 + C_1^7 (3xy^2)^6 (-z^2)^1 + C_2^7 (3xy^2)^5 (-z^2)^2$$
$$+ C_3^7 (3xy^2)^4 (-z^2)^3 + C_4^7 (3xy^2)^3 (-z^2)^4 + C_5^7 (3xy^2)^2 (-z^2)^5$$
$$+ C_6^7 (3xy^2)^1 (-z^2)^6 + C_7^7 (3xy^2)^0 (-z^2)^7$$

ان كان هو

$$C_4^7 (3xy^2)^3 z^8 = \frac{7.6.5.4!}{4!\,3\times 2} \cdot (3^3 x^3 y^6) z^8 = 35 * 27 x^3 y^6 z^8$$

(33a)

كما يمكن ايجاد هذا الحد أو اي حد آخر عن طريق الصيغة التالية

$$P_r = C^n_{r-1}\ a^{n-r+1}\ b^{r-1} \quad \cdots (*)$$

$n = 7$

$a = 3xy^2$

$b = -z^2$

$$\therefore P_r = C^7_{r-1}\ (3xy^2)^{7-r+1}\ (-z^2)^{r-1}$$

$$= \frac{7!}{(r-1)!\,(8-r)!}\ (3xy^2)^{8-r}\ (-z^2)^{r-1}$$

نقوم برفع كل الاس $8-r$

$$= \frac{7!}{(r-1)!\,(8-r)!}\ 3^{8-r}\ x^{8-r}\ y^{16-2r}\ (-z^2)^{r-1}$$

المطلوب هو كل الذي يحوي على y^6

$$\therefore y^6 = y^{16-2r}$$

اذا تساوت الحدود تتساوى الاسس وعليه:

$$6 = 16 - 2r \Longrightarrow 2r = 16 - 6$$

$$\therefore r = 5$$

$$\therefore P_5 = C^7_4\ (3xy^2)^{8-5}\ (-z^2)^4$$

$$= 35 \times 27\ x^3 y^6 z^8$$

EX: (2)

A class contains 9 boys and 3 girls:

① In how many ways can the teacher choose a committee of 4?

② How many of them will contain at least one girl?

③ How many of them will contain exactly one girl?

Sol:

① $C_4^{12} = \frac{12!}{4!(12-4)!} = 495$

② $C_3^9 \cdot C_1^3 + C_2^9 C_2^3 + C_1^9 C_3^3 = \frac{9 \cdot 8 \cdot 7 \cdot 6!}{3 \times 2 \; 6!} \cdot \frac{8 \times 3}{1 \times 2}$

$+ \frac{9 \cdot 8 \cdot 7!}{2 \times 7!} \cdot \frac{3 \times 2}{2} + \frac{9 \cdot 8!}{1 \cdot 8!} =$

$252 + 108 + 9 = 369$

الطريقة الأخرى

$C_4^{12} - C_4^9 = 369$

عدد الأيمان التي لا تحوي على بنت

③ $C_3^9 \cdot C_1^3 = 252$

EX: (3)

A woman has 11 close friends. أصدقاء مقربين

① In how many ways can she invite 5 of them to dinner?

② In how many ways if two of the friends are married and will not attend separately?

③ In how many ways if two of them are not on speaking terms and will not attend together?

① $C_5^{11} = \frac{11 \cdot 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6!}{5 \cdot 4 \cdot 3 \cdot 2 \cdot 6!} = 462$

② $C_3^9 + C_5^9 = 210 = \frac{9 \cdot 8 \cdot 7 \cdot 6!}{3 \times 2 \cdot 6!} + \frac{9 \cdot 8 \cdot 7 \cdot 6 \cdot 5!}{5! \cdot 4 \cdot 3 \cdot 2} = 84 + 126 = 210$

③ $C_5^9 + C_4^9 + C_4^9 = 378$

(العميل A باقي) (العميل B باقي)

$126 + 2\left[\frac{9 \cdot 8 \cdot 7 \cdot 6 \cdot 5!}{4 \cdot 3 \cdot 2 \cdot 5!}\right] = 126 + 252 = 378$

EX: (3)

There are 10 points A, B, ... in a plane, no three on the same line.

① How many lines are determined by the points?

② How many of these lines do not pass through A or B?

③ How many triangles are determined by the points?

④ How many of these triangles contain the point A?

⑤ How many of these triangles contain the side AB?

sol:

① $C_2^{10} = 45 = \frac{10 \cdot 9 \cdot 8!}{2 \cdot 8!} = 45$

② $C_2^8 = \frac{8 \cdot 7 \cdot 6!}{2 \cdot 6!} = 28$

③ $C_3^{10} = \frac{10 \cdot 9 \cdot 8 \cdot 7!}{3 \times 2 \cdot 7!} = 120$

④ $C_2^9 = \frac{9 \cdot 8 \cdot 7!}{2 \times 7!} = 36$

⑤ $C_1^8 = \frac{8 \cdot 7!}{7!} = 8$

EX: (4)

Find the number of ways in which 5 Large books, 4 medium-size books and 3 small books can be placed on a shelf so that all books of the same size are together:

$5!\ 4!\ 3!\ 3! = 103680$

EX: (5)

$0, 1, 2, 3, 4, 5, 6, 7, 8, 9$

consider all positive integers with 3 different digits. (note that 0 cannot be the first digits)

① How many are greater than 700 ?

② How many are odd ?

③ How many are even?

④ How many are divisible by 5 ?

Sol:

④ $3 \cdot 9 \cdot 8 = 216$

$9, 8, 7$

نبدأ بالطرف المقيد عليها شروط

② $8 \cdot 8 \cdot 5 = 320$

عليها شرط — عليها شرط

③ $8 \cdot 8 \cdot 5 = 320$