# Lecture - 8 -

$n^2 - 5n + 6 = 42 \Rightarrow n^2 - 5n - 36 = 0$

$(n-9)(n+4) = 0$

either $n - 9 = 0 \Rightarrow \boxed{n = 9}$

or $n + 4 = 0$ يهمل

(3) $2P(n,2) + 50 = P(2n,2)$

$$2 * \frac{n!}{(n-2)!} + 50 = \frac{(2n)!}{(2n-2)!}$$

$$2 * \frac{n(n-1)\cancel{(n-2)!}}{\cancel{(n-2)!}} + 50 = \frac{2n(2n-1)\cancel{(2n-2)!}}{\cancel{(2n-2)!}}$$

$2n^2 \cancel{-2n} + 50 = 4n^2 \cancel{-2n}$

$2n^2 - 50 = 0$

$n^2 - 25 = 0$

$(n-5)(n+5) = 0$

$n - 5 = 0 \Rightarrow \boxed{n = 5}$

$n + 5 = 0$ يهمل

## (3) Combinations : التوافيق

A combinations is a selection of objects without regard (بغض النظر) to their order, denoted by $C(n,r)$

التوافيق هي اختيار الاشياء بغض النظر عن ترتيبها.

Suppose we have a collection of $n$ objects. A combination of these $n$ objects taken $r$ at a time or an $r$-combination, is any subset of $r$ elements. In other words, an $r$-combination is any selection of $r$ of the $n$ objects where order does not count, and denoted by $C(n,r)$ or $\binom{n}{r}$ or $C^n_r$.

$$\therefore C_r^n = \frac{n!}{r!\,(n-r)!} = \frac{P(n,r)}{r!}$$

EX: (1)

How many committees of 3 can be formed from 8 people? Each committee is essentially a combination of the 8 people taken 3 at a time. Thus

$$C_3^8 = \frac{8!}{3!(8-3)!} = \frac{8!}{3!\,5!} = \frac{8.7.6.\cancel{5!}}{\cancel{3 \times 2} \times \cancel{5!}} = 56$$

different committees can be formed.

Remarks:

① If $r > n$ then $C_r^n = 0$

② If $r = n$ then $C_r^n = 1$

③ If $r = 0$ then $C_0^n = 1$

④ $C_r^n = C_{n-r}^n$

⑤ The number of combinations of $n$ objects of which $n_1$ are like, $n_2$ are like, ..., $n_r$ are like, is

$$C_{n_1, n_2, \ldots, n_r}^n = \frac{n!}{n_1!\, n_2! \cdots n_r!}, \quad n = n_1 + n_2 + \cdots + n_r$$

EX: (2)

The 3 men be chosen from the 7 men and the 2 women can be chosen from the 5 women, hence the committee can be chosen in:

$$C_3^7 \cdot C_2^5 = \frac{7!}{3!\,(7-3)!} \cdot \frac{5!}{2!\,(5-2)!} = \frac{7.\cancel{6}.5.\cancel{4!}}{\cancel{3 \times 2} * \cancel{4!}} \cdot \frac{5 * \overset{2}{\cancel{4}} * \cancel{3!}}{\cancel{2} * \cancel{3!}}$$

$$= 350$$

EX: (3)

A student is to answer 8 out of 10 questions on an example

① How many choies has he? $C^{10}_{8}$

② How many if he must answer the first 3 questions?

③ How many if he must answer at least 4 of the first 5 questions?

④ If he answer all the first 5 questions?

Sol:

① $C^{10}_{8} = \frac{10!}{8!\,(10-8)!} = \frac{\overset{5}{\cancel{10}}\cdot 9\cdot \cancel{8!}}{\cancel{8!}\cdot \cancel{2}!} = 45$ ways

② $C^{3}_{3}\cdot C^{7}_{5} = (1)\times \frac{7!}{5!\,2!} = \frac{7\times \overset{3}{\cancel{6}}\times \cancel{5!}}{\cancel{5!}\cdot \cancel{2}} = 21$ ways

③ $C^{5}_{4}\, C^{5}_{4} + C^{5}_{5}\, C^{5}_{3} = \frac{5!}{4!\,1!}\cdot\frac{5!}{4!\,1!} + (1)\,\frac{5!}{3!\cdot 2} = 35$

④ $C^{5}_{5}\cdot C^{5}_{3} = (1)\frac{5!}{3!\,2!} = 10$ ways

EX: (4)

Assume that there are $n=7$ students and that we wish to form 3 groups, 2 in the first, 3 in the second and 2 in the third? ~~let~~

Sol:

Let $n_1=2$, $n_2=3$ and $n_3=2$

$\Rightarrow n = n_1+n_2+n_3 = 2+3+2 = 7$

$C^{7}_{2}\cdot C^{5}_{3}\cdot C^{2}_{2} = \frac{7!}{2!\,5!}\cdot\frac{5!}{3!\,2!}\cdot\frac{\cancel{2!}}{\cancel{2!}}$

$= \frac{7\cdot \overset{3}{\cancel{6}}\cdot \cancel{5!}}{\cancel{2}\cdot \cancel{5!}}\cdot\frac{5\times \overset{2}{\cancel{4}}\cdot \cancel{3!}}{\cancel{3!}\,\cancel{2}}\cdot(1) = 210$

## * Combinations and Binomial theorem:

التوافيق ونظرية ثنائي الحدين

If $n$ is positive integer then:

$$(a+b)^n = \sum_{r=0}^{n} C_r^n a^r b^{n-r}$$

or $$= \sum_{r=0}^{n} C_r^n a^{n-r} b^r$$

$$(a+b)^n = \sum_{r=0}^{n} C_r^n a^r b^{n-r}$$

$$= C_0^n a^0 b^{n-0} + C_1^n a^1 b^{n-1} + C_2^n a^2 b^{n-2} + \dots + C_n^n a^n b^{n-n}$$

$$= b^n + n a b^{n-1} + \frac{n(n-1)}{2} a^2 b^{n-2} + \dots + a^n$$

$n=0 : (a+b)^0 = 1$

$n=1 : (a+b)^1 = a + b$

$n=2 : (a+b)^2 = a^2 + 2ab + b^2$

$n=3 : (a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$

$n=4 : (a+b)^4 = a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$

$n=5 : (a+b)^5 = a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + b^5$

خواص مفكوك ذي الحدين $(a+b)^n$

① مجموع أس كل من a و b في كل حد يساوي (n).

② يتناقص أس a حداً بعد حد من n الى الصفر ويتزايد أس الحد (b) من الصفر الى n حداً بعد حد.

③ معامل كل حد هو $C_r^n$ حيث ان (r) هو أس أي من الحدين a أو b.

④ عدد الحدود الناتجة يساوي (n+1)، يعني اذا كانت $n=3$ فان عدد الحدود الناتجة $= n+1=4$.