

* Theorem (1)

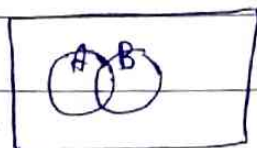
If $A \subset B$ then $P(A) \leq P(B)$

Proof:

$$B = A \cup (B \cap A')$$

$$P(B) = P(A \cup (B \cap A'))$$

$$P(B) = \underbrace{P(A)}_{\text{م.ع.}} + P(B \cap A')$$



$$\therefore P(B) \geq P(A)$$

* Theorem (2)

For any two events A and B then

$$P(A \setminus B) = P(A) - P(A \cap B)$$

Proof:

$$A = (A \setminus B) \cup (A \cap B)$$

$\rightarrow \text{م.ع.}$

$$P(A) = P((A \setminus B) \cup (A \cap B))$$

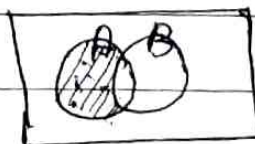
$$P(A) = P(A \setminus B) + P(A \cap B)$$

$$\Rightarrow P(A \setminus B) = P(A) - P(A \cap B)$$

نلاحظ ان $A \setminus B$ هي المنطقة التي لا تحتوي على $A \cap B$

$$A \setminus B = A \cap B'$$

$$B \setminus A = B \cap A'$$



$B \setminus A$ هي المنطقة التي لا تحتوي على $A \cap B$

Proof:

$$A = (A \cap B') \cup (A \cap B)$$

$$P(A) = P((A \cap B') \cup (A \cap B))$$

$\rightarrow \text{م.ع.}$

$$P(A) = P(A \cap B') + P(A \cap B)$$

$$\Rightarrow P(A \cap B') = P(A) - P(A \cap B)$$

$$\therefore A \setminus B = A \cap B'$$

$$\Rightarrow P(A \setminus B) = P(A \cap B')$$

$$= P(A) - P(A \cap B)$$

EX: (1)

Two coins are tossed what is the probability that at least one head appears?

sol:

$$S = \{HH, HT, TH, TT\}$$

$$A = \{\text{at least one head}\} = \{HH, HT, TH\}$$

$$P(A) = \frac{n}{N} = \frac{n(A)}{n} = \frac{3}{4} = \frac{C_1^3}{C_1^4}$$

$$B = \{\text{exactly one is head}\} = \{HT, TH\}$$

$$P(B) = \frac{2}{4} \text{ or } \frac{C_1^2}{C_1^4} = \frac{2}{4} = 0.5$$

$$C = \{\text{exactly two head}\} = \{HH\}$$

$$P(C) = \frac{1}{4} = \frac{C_1^1}{C_1^4} = \frac{1}{4}$$

$$A \cap B = \{HT, TH\}$$

$$P(A \cap B) = \frac{2}{4} = \frac{C_1^2}{C_1^4}$$

EX: (2)

EX: (8)

A class contains 10 men and 20 women of which half the men and half women have brown eyes. Find the probability that a person chosen at random is a man or has brown eyes.

Sol: المطلوب هو احتمال اختيار شخص أو امرأة ويكون رجل أو يملك عيون بني.

$A = \{ \text{person is a man} \}$

$B = \{ \text{person has brown eyes} \}$

$$n(S) = C_1^{30} \text{ اختيار شخص واحد من 30 شخصاً}$$

$$n(S) = 30$$

$$P(A) = \frac{n(A)}{n(S)} = \frac{C_1^{10}}{C_1^{30}} = \frac{10}{30} = \frac{1}{3}$$

$$P(B) = \frac{n(B)}{n(S)} = \frac{C_1^{15}}{C_1^{30}} = \frac{15}{30} = \frac{1}{2}$$

15 = 5 + 10

$$P(A \cup B) = \frac{n(A \cup B)}{n(S)}$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$A \cap B = \{ \text{person is man and have brown eyes} \}$

$$P(A \cap B) = \frac{C_1^5}{C_1^{30}} = \frac{5}{30} = \frac{1}{6}$$

$$\therefore P(A \cup B) = \frac{1}{3} + \frac{1}{2} - \frac{1}{6} = \frac{2+3-1}{6} = \frac{4}{6} = \frac{2}{3}$$

Find the probability that a person chosen at random is woman and have brown eyes.

$A = \{ \text{person is woman and have brown eyes} \}$

$$P(A) = \frac{C_1^{10}}{C_1^{30}} = \frac{10}{30} = \frac{1}{3}$$

EX: If $S = \{a, b, c, d\}$, which statements defines a probability space on S from the following?

أي من العبارات التالية لا تعرف فضاء احتمالي على S ؟

① $P(a) = \frac{1}{2}$, $P(b) = \frac{1}{3}$, $P(c) = \frac{1}{4}$ and $P(d) = \frac{1}{5}$

② $P(a) = \frac{1}{8}$, $P(b) = P(c) = \frac{1}{4}$ and $P(d) = \frac{1}{3}$

③ $P(a) = P(c) = \frac{1}{3}$, and $P(b) = P(d) = \frac{1}{6}$

④ $P(a) = 0.2$, $P(b) = 0.5$, $P(c) = 0.3$ and $P(d) = 0.4$

Sol:

$P(a) + P(b) + P(c) + P(d) = 1$ مجموع الاحتمالات يجب ان

$\frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} = \frac{30+20+15+12}{60}$ باي واحد

$= \frac{77}{60} = 1.28 > 1$

∴ هذه الاحتمالات لا تعرف فضاء احتمالي على S .

3.5 Independent Events الاستقلال

Two events A and B are independent iff:

$P(A \cap B) = P(A) \cdot P(B)$

Two events A and B are considered to be independent, if the occurrence (or non occurrence) of one does not affect the probability assigned to the occurrence of the other.

اذا حدثت A و B يتغيران فقلان

EX: (1) اذا كان ظهر، آردعك ظهر، ادرهما لا ياتر على احتمال كل واحد

suppose that two machines 1 and 2 are operated in a factory independently of each other. Let A be the event that machine 1 will become inoperative during a given 8-hours period, Let B be the event