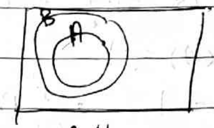
Lecture -11-

theorems:

1) If $A \subset B$ then $P(A) \leq P(B)$

Proof



A U

$P(A) \leq P(B)$

$B = A \cup (B \cap A')$

$P(B) = P(A \cup (B \cap A'))$

$P(B) = P(A) + P(B \cap A')$

If $P(B \cap A') > 0$ Then

$P(B) > P(B \cap A')$

2) $P(A \setminus B) = P(A) - P(A \cap B)$

3) $P(A \cap B') = P(A) - P(A \cap B)$

4) $P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B)$

$- P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$

EX: (3)

Let 2 items be chosen at random from a

علبة. مجموعة أشياء

Lot containing 12 items of which 4 are defective. معيوبة

Let

$A = \{$ both items are defective $\}$

$B = \{$ both items are non-defective $\}$

$C = \{$ one is defective and one is non-defective $\}$

Find probability of the above events.

$n(S) = C^{12}_{2} = \frac{12!}{2!\,(10!)} = \frac{\overset{6}{\cancel{12}} \cdot 11 \cdot \cancel{10!}}{\underset{1}{\cancel{2}}\ \cancel{10!}} = 66$

12 items
4 defective, 8 non-defective

$n(A) = C^{4}_{2} = \frac{\cancel{4} \cdot 3 \cdot 2!}{\cancel{2} \cdot 2} = 6$

$n(B) = C^{8}_{2} = \frac{\overset{4}{\cancel{8}} \cdot 7 \cdot \cancel{6!}}{\cancel{2} \cdot \cancel{6!}} = 28$

$n(C) = C^{4}_{1} \cdot C^{8}_{1} = \frac{4 \cdot \cancel{3!}}{\cancel{3!}} \cdot \frac{8 \cdot \cancel{7!}}{\cancel{7!}} = 32$

$\therefore P(A) = \frac{n(A)}{n(S)} = \frac{6}{66} = \frac{1}{11}$

$P(B) = \frac{28}{66}$

$P(C) = \frac{32}{66}$

$D = \{$ at least one defective $\}$

$E = \{$ at least one non-defective $\}$

$F = \{$ at most one defective $\}$

$G = \{$ at most one non-defective $\}$

EX: (4)

A pair of dice is to be tossed once. Find the probability of the following events.

① $E_1 = \{$ The sum of the two face is 7 $\}$
② $E_2 = \{$ The sum of the two face is 11 $\}$
③ $E_3 = \{$ The sum of the two face is either 7 or 11.
④ $E_3 = \{$ The first face is greater than second face $\}$
⑤ $E_4 = \{$ The difference of two faces is zero $\}$
⑥ $E_5 = \{$ The second face shows even number $\}$
⑦ $E_6 = \{$ The 1-st is dice shows ~~one~~ an number (3) $\}$
⑧ $E_7 = \{$ The 2nd dice shows number 1 or 5 and the sum of the two number is less than 8 and more than 3 $\}$

Sol: $n(S) = $

	1	2	3	4	5	6
1	(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)
2	(2,1)	(2,2)	(2,3)	(2,4)	(2,5)	(2,5)
3						
4						
5						

$E_1 = \{(1,6), (6,1), (2,5), (5,2), (3,4), (4,3)\}$

$E_2 = \{(5,6), (6,5)\}$, $P(E_2) = \frac{2}{36}$

$P(E_1) = \frac{6}{36}$

$E_3 = \{(1,6), (6,1), (2,5), (5,2), (3,4), (4,3), (5,6), (6,5)\}$

$P(E_3) = \frac{8}{36}$ or $P(E_1 \cup E_2) = \frac{8}{36}$

EX: (5)

Suppose a sample space S consists of 4 elements $S = \{a_1, a_2, a_3, a_4\}$. which function defines a probability space on S?

على فرض ان فضاء العينة S يتكون من (4) نقاط.

ماهي الدالة التي تعرف احتمال الفضاء S (معطيا ياتي)

1) $P(a_1) = \frac{1}{2}$, $P(a_2) = \frac{1}{3}$, $P(a_3) = \frac{1}{4}$, $P(a_4) = \frac{1}{5}$

② $P(a_1) = \frac{1}{2}$, $P(a_2) = \frac{1}{4}$, $P(a_3) = \frac{-1}{4}$, $P(a_4) = \frac{1}{2}$

③ $P(a_1) = \frac{1}{2}$, $P(a_2) = \frac{1}{4}$, $P(a_3) = \frac{1}{8}$, $P(a_4) = \frac{1}{8}$

④ $P(a_1) = \frac{1}{2}$, $P(a_2) = \frac{1}{4}$, $P(a_3) = \frac{1}{4}$, $P(a_4) = 0$

تكون الدالة (اي نقطة من النقاط الاربعة السابقة) تعرف احتمال فضاء العينة عند ما تحقق الشروط الآتية

$P(a_i) \geq 0 \quad , i=1,2,3,4$

$\sum_{i=1}^{4} P(a_i) = 1$

① نلاحظ ان $P(a_i) \geq 0$ (اي موجبة) وان

$P(a_1)+P(a_2)+P(a_3)+P(a_4) = \frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{5} = \frac{60+40+30+24}{120} = \frac{154}{120}$

$= \frac{77}{60} > 1$

∴ الدالة لا تعرف احتمال الفضاء (S).

② نلاحظ في هذه الفقرة ان $P(a_3) = -\frac{1}{4} < 0$ اي لم تحقق الشرط الاول مما يؤدي الى انها لا تمثل احتمال الفضاء S.

③ بـ $P(a_i) \geq 0$ وان $\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{8} = \frac{4+2+1+1}{8} = \frac{8}{8} = 1$

∴ الدالة هنا تعرف احتمال الفضاء S.

④ ∴ $P(a_i) \geq 0$ وان $\frac{1}{2}+\frac{1}{4}+\frac{1}{4}+0 = 1$

∴ الدالة تعرف احتمال الفضاء S.

EX: (6)

A bag contains 8 white and 6 red balls, 4 balls are drawn at random from this bag, finds the probability of the following events:

A = {there will be 2 at red balls} كرتان حمراء

B = {at least 2 white} على الاقل 2 بيضاء

C = {at most 2 red} على الاكثر 2 حمراء

Sol.:

$8\ W \ , \ 6\ R \ , \ r = 4 \ , \ n = 8+6 = 14$

$n(S) = C^{14}_{4} = \frac{14!}{4!\,10!} = \frac{14.13.12.11.10!}{4.3.2.10!} = 1001$

$n(A) = C^{6}_{2} \cdot C^{8}_{2} = \frac{6.5.4!}{2.4!} \times \frac{8.7.6!}{2.6!} = 15 \times 28 = 420$

$\therefore P(A) = \frac{n(A)}{n(S)} = \frac{420}{1001}$

$$P(B) = \frac{C_2^8 C_2^6 + C_3^8 C_1^6 + C_4^8 C_0^6}{1001}$$

B: تشمل على الأقل 2 بيضاء
فممكن ان تكون 2 أو 3 أو 4 بيضاء

$$= \frac{420 + 56 + 70}{1001} = \frac{546}{1001}$$

$$P(C) = \frac{C_2^6 C_2^8 + C_1^6 C_3^8 + C_0^6 C_4^8}{1001}$$

C: على الأكثر 2 حمراء
فاما 2 أو 1 أو صفر

$$P(C) = \frac{546}{1001}$$