Example 3.2 Generate OR function using McCulloch-Pitts neuron model.

Solution The OR function returns a high ('1') if any one of the input is high, returns a low ('0') if none of the inputs is high.

The truth table for OR function is,

x_1	x_2	y
1	1	1
1	0	1
0	1	1
0	0	0

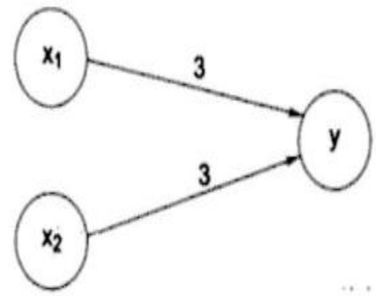

Fig. 3.3 McCulloch-Pitts Neuron for OR Function

A McCulloch-Pitts neuron for OR functions is shown in Fig. 3.3. The threshold for the unit is 3.

The net input is calculated as,

$$y_{in} = 3x_1 + 3x_2$$

The output is given by,

$$y = f(y_{in}) = \begin{cases} 1 & \text{if} \quad y_{-in} \geq 3 \\ 0 & \text{if} \quad y_{-in} < 3 \end{cases}$$

Presenting the inputs,

(i) $x_1 = x_2 = 1$, $y_{in} = 3x_1 + 3x_2 = 1 + 1 = 2$

$= 3 \times 1 + 3 \times 1 = 6 >$ threshold 3.

Hence, $y = 1$

(ii) $x_1 = 1$, $x_2 = 0$,

$y_{in} = 3x_1 + 3x_2$

$= 3 \times 1 + 3 \times 0 = 3 =$ threshold

Applying activation formula

$y = f(y_{in}) = 1$

This is also the case when $x_1 = 0$ and $x_2 = 1$

(iii) $x_1 = x_2 = 0$,

$y_{in} = 3x_1 + 3x_2 = 3 \times 0 + 3 \times 0 = 0 <$ threshold

Hence output $y = 0$.

Example 3.3 Realize NOT function using McCulloch-Pitts neuron model.

Solution The NOT function returns a true value ('1') if the input is false ('0') and returns a false value ('0') if the input is true ('1').

The truth table for NOT function is,

x	y
1	0
0	1

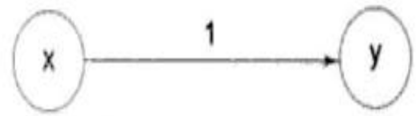

Fig. 3.4 *McCulloch-Pitts Neuron for NOT Function*

The McCulloch-Pitts neuron for this function is given in Fig. 3.4. The threshold for unit y is 1.

The net input is,

$$y_{in} = x \cdot w$$

Since $w = 1, y_{in} = x$.

The output activation is given by,

$$y = f(y_{in}) = \begin{cases} 1 & \text{if} \quad y_{-in} < 1 \\ 0 & \text{if} \quad y_{-in} \geq 1 \end{cases}$$

Presenting the input,

(i) $x_1 = 1, \quad y_{in} = 1$

Applying activation,

$$y = f(y_{in}) = 0$$

(ii) $x_1 = 0, \; y_{in} = 0$

Applying activation

$$y = f(y_{in}) = 1.$$

Example 3.4 Generate the output of ANDNOT function using McCulloch-Pitts neuron.

Solution The ANDNOT function returns a true value ('1') if the first input value is true ('1') and the second input value is false ('0').

The truth table for the ANDNOT functions is,

x_1	x_2	y
1	1	0
1	0	1
0	1	0
0	0	0

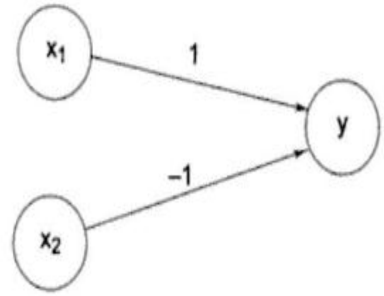

Fig. 3.5 *McCulloch-Pitts Neuron for ANDNOT Function*

The McCulloch-Pitts neuron for the ANDNOT function is shown in Fig. 3.5. The threshold of unit Y is 1.

The net input is,

$$y_{in} = x_1w_1 + x_2w_2$$
$$= x_1 * 1 + x_2 * -1$$
$$y_{in} = x_1 - x_2$$

The output activation is given as,

$$y = f(y_{in}) = \begin{cases} 1 & \text{if} \quad y_{-in} \geq 1 \\ 0 & \text{if} \quad y_{-in} < 1 \end{cases}$$

Presenting the input

(i) $x_1 = x_2 = 1,\quad y_{in} = x_1 - x_2 = 1 - 1 = 0 < 1$

Hence, $\quad y = f(y_{in}) = 0$

(ii) $x_1 = 1, x_2 = 0,\quad y_{in} = x_1 - x_2 = 1 - 0 = 1 = 1$

$y = f(y_{in}) = 1$

(iii) $x_1 = 0, x_2 = 1,\quad y_{in} = x_1 - x_2 = 0 - 1 = -1 < 1$

$y = f(y_{in}) = 0$

(iv) $x_1 = x_2 = 0,\quad y_{in} = x_1 - x_2 = 0 - 0 = 0 < 1$

$y = f(y_{in}) = 0.$

Thus AND NOT function is realized.

Example 3.5 Realize the Exclusive-OR function using McCulloch-Pitts neuron.

Solution XOR function returns a true value if exactly one of the input values is true; otherwise it returns the response as false. The truth table for XOR function is,

x_1	x_2	y
1	1	0
1	0	1
0	1	1
0	0	0

The McCulloch-Pitts neuron model for this is given in Fig 3.6. The threshold of unit y is 1.

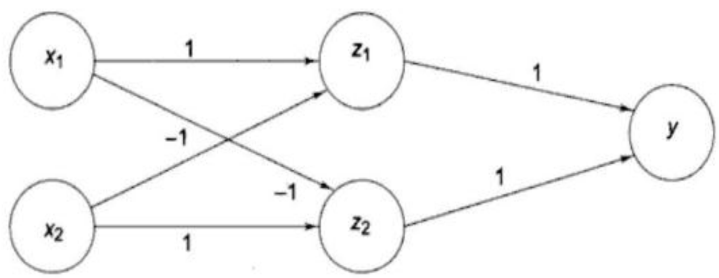

Fig. 3.6 *McCulloch-Pitts Neuron for XOR Function*

With one layer alone, it was not able to predict the value of the threshold for the neuron to fire, hence another layer is introduced.

$$x_1 \text{ XOR } x_2 = (x_1 \text{ ANDNOT } x_2) \text{ OR } (x_2 \text{ ANDNOT } x_1)$$

$$x_1 \text{ XOR } x_2 = z_1 \text{ OR } z_2$$

where, $z_1 = x_1 \text{ ANDNOT } x_2$

and $z_2 = x_2 \text{ ANDNOT } x_1$

The activations of z_1 and z_2 are given as,

$$z_1 = (z_{in-1}) = \begin{cases} 1 & \text{if} \quad z_{in-1} \geq 1 \\ 0 & \text{if} \quad z_{in-1} < 1 \end{cases}$$

$$z_2 = (z_{in-2}) = \begin{cases} 1 & \text{if} \quad z_{in-2} \geq 1 \\ 0 & \text{if} \quad z_{in-2} < 1 \end{cases}$$

The calculation of net input and activations of z_1 and z_2 are shown below.

$z_1 = (x_1 \text{ ANDNOT } x_2)$ $\quad z_{in-1} = x_1w_1 + x_2w_2$

$w_1 = 1, w_2 = -1$

x_1	x_2	z_{in-1}	z_1
1	1	0	0
1	0	1	1
0	1	−1	0
0	0	0	0

$z_2 = (x_2 \text{ ANDNOT } x_1)$ $\quad z_{in-2} = x_1w_1 + x_2w_2$

$w_1 = -1, w_2 = 1$

x_1	x_2	Z_{in-2}	z_2
1	1	0	0
1	0	−1	0
0	1	1	1
0	0	0	0

The activation for the output unit y is 1.

$$y = f(y_{in}) = \begin{cases} 1 & \text{if} \quad y_{in} \geq 1 \\ 0 & \text{if} \quad y_{in} < 1 \end{cases}$$

Presenting the input patterns (z_1 and z_2) and calculating net input and activations gives output of XOR.

Here, $y_{in} = z_1w_1 + z_2w_2$

$w_1 = 1, w_2 = 1$

z_1	z_2	y_{in}	$y = z_1$ or z_2
0	0	0	0
1	0	1	1
0	1	1	1
0	0	0	0

Thus the Exclusive-OR function is realized.