Lecture -10-

Chapter three: Probability:

3-1: Introduction:

Probability is the study of random or nondeterministic experiments (that is, we wish to consider some experiment results and the repetition of the experiment does not always produce the same results).

هي دراسة لتجربة عشوائية أو غير محددة (ذلك انه نرغب في ان نرى بعض نتائج التجربة وان التكرار التجربة لا يعطي دائماً نفس النتائج).

3-2: Random experiment, Sample space and Event:

1- Random experiment:

Is a procedure which results in some nondeterministic outcomes in a particular situation.

وهي الاسلوب الذي ينتج مخرجات غير محددة في حالة معينة.

2- Sample space:

The set of all possible outcomes of a random experiment, denoted by S or Ω.

وهي مجموعة كل المخرجات الممكنة لتجربة عشوائية ويرمز لها ب S أو Ω

3- Event:

Is a subset of sample, denoted by E,

وهي مجموعة جزئية من فضاء التجربة (العينة) يرمز لها بالرمز E.

4- Elementary Event:

Is an event consisting of only one element of the sample space S.

هي حادثة تتكون من عنصر واحد فقط موجودة في فضاء الحالة

EX:(1)

If the experiment consists of tossing a coin once, then

$S = \{H, T\}$, $n = 1$ عدد مرات تكرار التجربة

∴ عدد النتائج الكلية الممكنة - لرمي قطعة النقود مرة واحدة تساوي عدد نتائج التجربة مرفوع للأس n

$2^n = 2^1 = 2$

If the experiment consists of tossing twice, then

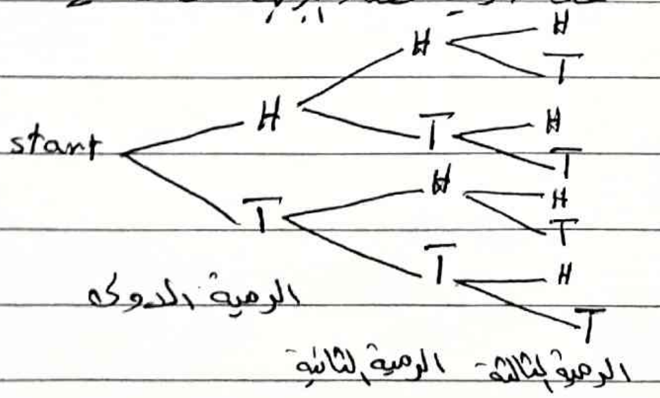
$S = \{HH, HT, TH, TT\}$, $n = 2$

$2^2 = 4$ عدد النتائج الكلية الممكنة (عدد عناصر مجموعة فضاء التجربة)

If the experiment consists of tossing three times, then

$S = \{HHH, HHT, HTH, THH, HTT, THT, TTH, TTT\}$

$2^3 = 8$ عناصر في فضاء التجربة



HHH
HHT
HTH
HTT
THH
THT
TTH
TTT

EX:(2)

If a couple ~~is planning to~~ have 3 children then

$S = \{GGG, GGB, GBG, BGG, GBB, BGB, BBG, BBB\}$

If E is the event that the couple have 2 girl then:

$E = \{GGB, GBG, BGG\}$

3.3 The classical approach

The first Low In probability

Suppose an event E can happen in (h) ways out of a total of n possible equally Like ways. Then the probability of occurrence of the event (called it success) is denoted by:

على فرض أن الحادثة E يمكن أن تحدث في (h) من الطرائق من عدد
(n) من الطرائق المتشابهة المتساوية الممكنة ،
بذلك فإن احتمالية حدوث الحادثة (تسمى بالنجاح) يرمز لها ب :

$$P = P(E) = \frac{h}{n} \quad \frac{\text{الجزء}}{\text{الكل}} \quad = \frac{n(E)}{n}$$

That means: The probabity of event E equals the number of outcomes favourablle (ايجابية) to E divided by the total number of outcomes,

احتمالية حدوث الحادثة E تساوي عدد المخرجات الايجابية (الناجحة)
عدد E مقسومة على جميع المخرجات الناتجة من التجربة ،

i.e.

$$P(E) = \frac{\text{favourable outcomes}}{\text{Total outcomes}}$$

The probability of non-occurence of the event (called its failure) is denoted by

$$q = P(\text{not } E) = \frac{n-h}{n}$$

or

$$q = 1 - \frac{h}{n} = 1 - P = 1 - P(E)$$

Thus $p + q = 1$

or $P(E) + P(\text{not } E) = 1$

EX: (1)

A ~~fair~~ coin is tossed once, what is the probability of getting the head?

$S = \{H, T\}$

$E = \{H\}$

$P(E) = \frac{1}{2}$

EX: (2)

when an unbiased dice is thrown once, ~~then~~ what is the probability of getting: the even number, the Prime number,

sol:

The six outcomes are mutually exclusive because two or more faces cannot ~~happen~~ occur ~~appear~~ simultaneously then

$P(1) = P(2) = P(3) = P(4) = P(5) = P(6) = \frac{1}{6}$

$S = \{1, 2, 3, 4, 5, 6\}$

$E_1 = \{\text{even number}\}$

$\therefore E_1 = \{2, 4, 6\}$

$P(E_1) = \frac{3}{6} = \frac{1}{2}$

$E_2 = \{\text{Prime number}\}$

$\therefore E_2 = \{2, 3, 5\}$

$P(E_2) = \frac{3}{6} = \frac{1}{2}$

3.4 Axioms of probability : بديهيات الاحتمال

If A is any event then.

1) $0 \leq P(A) \leq 1$

2) $P(S) = 1$, $\sum_{i=1}^{n} P_i = 1$

3) If A and B are mutually exclusive events, Then

a) $P(A \cup B) = P(A) + P(B)$,

b) If A and B are not mutually exclusive or joint events, then $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

4) If $A_1, A_2, \ldots$ is sequence of mutually exclusive events, then

$P(A_1 \cup A_2 \cup \cdots) = P(A_1) + P(A_2) + \cdots$

and if A_i is sequence of mutually exclusive ~~and~~ when $i = 1, 2, \ldots, n$ Then

$P(A_1 \cup A_2 \cup \cdots \cup A_n) = P(A_1) + P(A_2) + \cdots + P(A_n)$

5) $P(\phi) = 0$

Prove: Let A be any set, then A and ϕ are disjoint and $A \cup \phi = A$

$\Rightarrow \cancel{P(A \cup \phi) = P(A}$

$\Rightarrow P(A) = P(A \cup \phi) = P(A) + P(\phi)$

subtracting $P(A)$ from both sides. then

$P(\phi) = 0$

6) If A' is the complement of an event A, then

$P(A') = 1 - P(A)$

Prove: The sample space S can be decomposed تتحلل into The mutually exclusive events A and A' that is:

$S = A \cup A'$, by (2) and (3) we obtain

$P(S) = 1 \Rightarrow P(A \cup A') = 1 = P(A) + P(A')$

$\Rightarrow P(A') = 1 - P(A)$