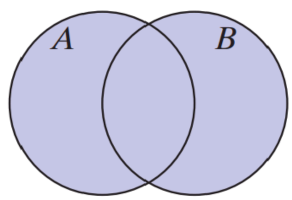
1-Union and Intersection

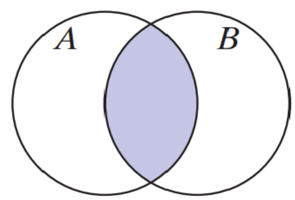
The union of the sets A and B denoted by $A \cup B$ is the set of elements that belong the set A or to set B or to both sets A and B .

The Intersection of the sets A and B denoted by $A \cap B$ is the set of elements that common to both sets A and B .

The concepts of the union and intersection of two sets are illustrated in figures :



$A \cup B$
A union *B*
The elements in *A or B or* both



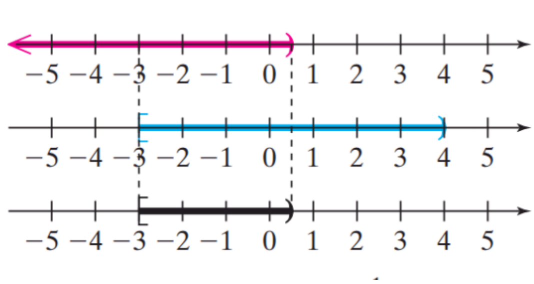
$A \cap B$
A intersection *B*
The elements in *A and B*

Example 1 :

Find the intersection of : $\left(-\infty , \frac{1}{2}\right) \cap (-3 , 4)$

Solution :

To find the intersection , graph each intervals separately , then find the real numbers common to both intervals



$\left(-\infty, \frac{1}{2}\right)$

$[-3, 4)$

The intersection is the "overlap" of the two intervals: $[-3, \frac{1}{2})$

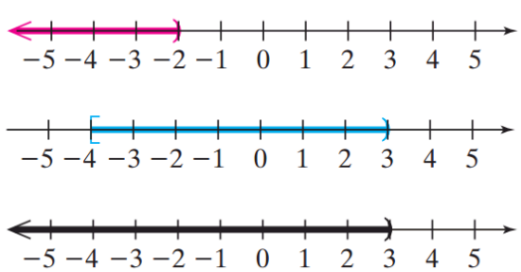
The intersection is $[-3\,,\frac{1}{2})$

Example 2 :

Find the union of : $(-\infty\,,-2\,)\cup(-4\,,3)$

Solution :

To find the union , graph each intervals separately , the union is the collection of real numbers that lie in the 1st interval the 2nd interval , or both interval .



$(-\infty, -2)$

$[-4, 3)$

The union consists of all real numbers in the red interval along with the real numbers in the blue interval: $(-\infty, 3)$

2- Solving compound Inequalities : and - or

The solution to two inequalities joined by the word " and " is the intersection of their solution sets , the solution of two inequalities joined by the word " or " is the union of their solution sets .

Summary :

- Solve and graph each inequality separately .
- if the inequality are joined by the word " and " find the intersection of the two solution sets .
- if the inequality are joined by the word " or " find the union of the two solution sets .
- Express the solution set in interval notation or in set builder notation .

Note :

Remember that multiplying or dividing an inequality by a negative factor reverses the direction of the inequality sign .

Example 3 :

Find the solution of compound inequality :

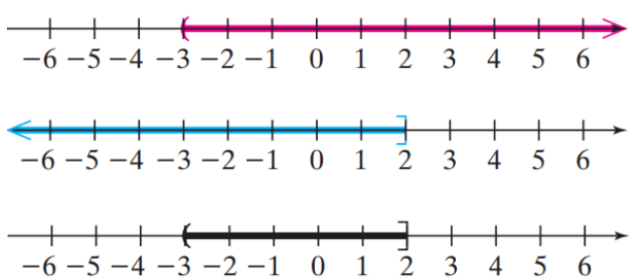
$$-2x < 6 \quad and \quad x + 5 \leq 7$$

Solution :

$-2x < 6 \quad and \quad x + 5 \leq 7$

$x > -3 \quad and \quad x \leq 7 - 5$

$x > -3 \quad and \quad x \leq 2$



$\{x \mid x > -3\}$

$\{x \mid x \leq 2\}$

Take the intersection of the solution sets: $\{x \mid -3 < x \leq 2\}$

Example 4 :

Find the solution of compound inequality :

$$-2 \leq -3x + 1 < 5$$

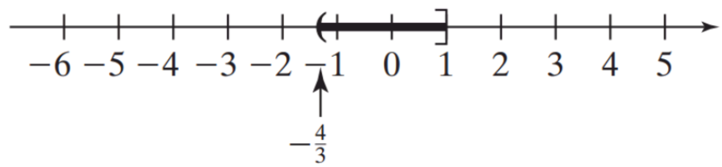
Solution :

$-2 \leq -3x + 1 \quad and \quad -3x + 1 < 5$

$-3 \leq -3x \quad and \quad -3x < 4$

$1 \geq x$ *and* $x > \frac{-4}{3}$

$\frac{-4}{3} < x \leq 1$



The solution is $(\frac{-4}{3}, 1]$

Example 5 :

Find the solution of compound inequality :

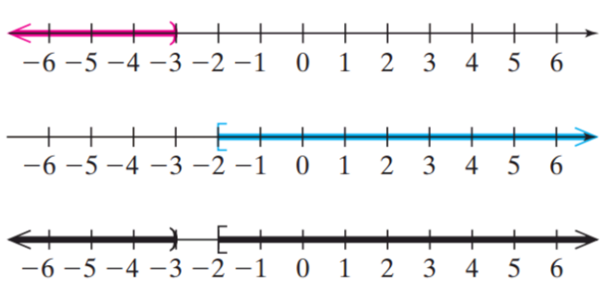
$$-3y - 5 > 4 \quad or \quad 4 - y \leq 6$$

Solution :

$-3y - 5 > 4 \quad or \quad 4 - y \leq 6$

$-3y > 9 \quad or \quad -y \leq 2$

$y < -3 \quad or \quad y \geq -2$



$\{y \mid y < -3\}$

$\{y \mid y \geq -2\}$

Take the union of the solution sets $\{y \mid y < -3$ or $y \geq -2\}$.

The solution is $(-\infty, -3) \cup [-2, \infty)$

3- Solving Inequality Graphically

Quadratic inequalities are inequalities that can be written in any of the following forms such that $a \neq 1$

$$ax^2 + bx + c \geq 0$$

$$ax^2 + bx + c > 0$$

$$ax^2 + bx + c \leq 0$$

$$ax^2 + bx + c < 0$$

The graph of the quadratic function defined by: $f(x) = ax^2 + bx + c, a \neq 1$

Is a parabola that opens upward or downward the quadratic inequality $f(x) > 0$ or equivalently $ax^2 + bx + c > 0$ is asking the questions " for what values of x is the value of the function positive (above the x-axis) " , the quadratic inequality $f(x) < 0$ or equivalently $ax^2 + bx + c < 0$ is asking the questions " for what values of x is the value of the function negative (below the x-axis) " ,the graph of the quadratic function can be used to answer these questions .

Notes :

- If $f''(x) > 0$ then the curve is concave
- If $f''(x) < 0$ then the curve is convex
- To graph any figure we will find the intersection point with x and y axes .

Example 6 :

Use the graph to solve the quadratic inequality :

$$f(x) = x^2 - 6x + 8$$

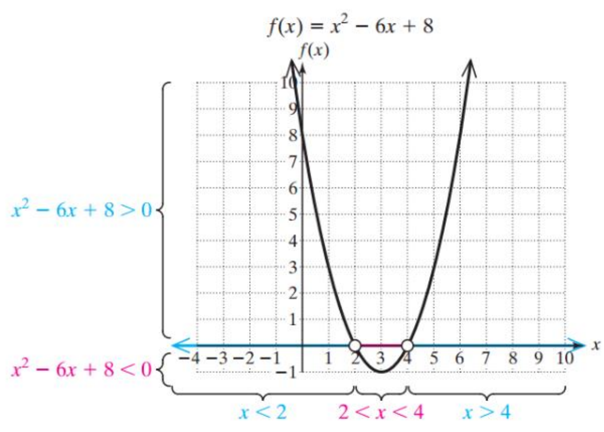
If : A) $f(x) < 0$, B) $f(x) > 0$

Solution :

$x = 0 \Rightarrow y = 8$

$y = 0 \Rightarrow 0 = x^2 - 6x + 8 \quad \Rightarrow 0 = (x-2)(x-4) \Rightarrow x = 2, 4$

The intersection points are $(0,8)$, $(2,0)$, $(4,0)$



A) The solution to $x^2 - 6x + 8 < 0$ is the set of real numbers x for which $f(x) < 0$ Graphically this is the set of all x values corresponding to the points where the parabola is below the x axis , then the solution of $f(x) < 0$ is :
$\{x : x \in R , 2 < x < 4\} = (2, 4)$

B) The solution to $x^2 - 6x + 8 > 0$ is the set of real numbers x for which $f(x) > 0$ Graphically this is the set of all x values corresponding to the points where the parabola is above the x axis , then the solution of $f(x) > 0$ is :
$\{x : x \in R , x < 2 \; or \; x > 4\} = (-\infty, 2) \cup (4, \infty)$

Example 7 :

Use the graph to solve the quadratic inequality :

$$f(x) = \frac{1}{x+1}$$

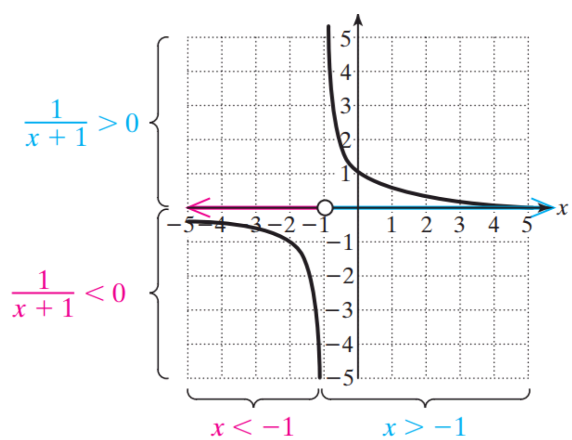
If : A) $f(x) < 0$, B) $f(x) > 0$

Solution :

$x = 0 \Rightarrow y = 1$

$y = 0 \Rightarrow no \; point$

The intersection points are $(0, 1)$, x = -1 is vertical asymptotic .



A) The solution to $\frac{1}{x+1} < 0$ is the set of real numbers x for which $f(x) < 0$ Graphically this is the set of all x values such that : $\{x : x \in R , x < -1\} = (-\infty, -1)$

B) The solution to $\frac{1}{x+1} > 0$ is the set of real numbers x for which $f(x) > 0$ Graphically this is the set of all x values such that : $\{x : x \in R , x > -1\} = (-1, \infty)$

4- Solving polynomial Inequality by using Test Point Method

Boundary point : the boundary points of an inequality consist of the real solutions to the related equation and the points where the inequality is undefined .

Testing points in regions bounded by these points is the basis of the test point method to solve inequalities .

Summary

- Find the boundary points of the inequality .
- Plot the boundary points on the number line , this divides the number line into regions .
- Select a test point from each region and substitute it into the original inequality .

- If the test point makes the original inequality true then that region is part of the solution set .
- Test the boundary points in the original inequality .
- If a boundary point makes the original inequality true then that point is part of the solution set .

Example 8 :

Use the test point method to solve the polynomial inequality :

A) $2x^2 + 5x < 12$, B) $x(x-2)(x-4)(x+4)^2 > 0$

Solution :

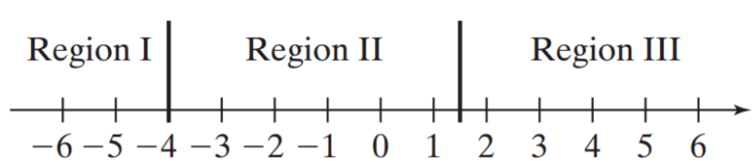
A)

$$2x^2 + 5x = 12$$

$$2x^2 + 5x - 12 = 0 \Longrightarrow (2x-3)(x+4) = 0$$

$\therefore x = \frac{3}{2} \ -4$ are boundary points

Now plot the boundary point in the number line :



Testing : Region 1 (x=-5)

$2x^2 + 5x < 12$

$2(-5)^2 + 5(-5) < 12$

$25 < 12$ is False

Testing : Region 2 (x=0)

$2x^2 + 5x < 12$

$2(0)^2 + 5(0) < 12$

$0 < 12$ is True

Testing : Region 3 (x=2)

$2x^2 + 5x < 12$

$2(2)^2 + 5(2) < 12$

$18 < 12$ is False

Testing : Boundary points ($x = \frac{3}{2}$, $x = -4$)

$2x^2 + 5x < 12$

$x = \frac{3}{2} \Rightarrow 2(\frac{3}{2})^2 + 5(\frac{3}{2}) < 12 \Rightarrow 12 < 12$ is False

$x = -4 \Rightarrow 2(-4)^2 + 5(-4) < 12 \Rightarrow 12 < 12$ is False

So the solution set is : $\left\{x : x \in R , -4 < x < \frac{3}{2}\right\} = (-4, \frac{3}{2})$

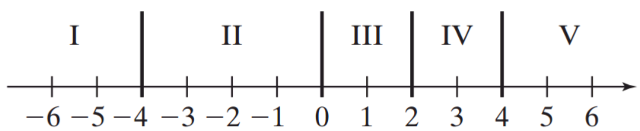
B)

$x(x-2)(x-4)(x+4)^2 > 0$

$x(x-2)(x-4)(x+4)^2 = 0$

$\therefore x = 0, 2, 4, -4$ are boundary points

Now plot the boundary point in the number line :



Testing : Region 1 (x=-5) : $x(x-2)(x-4)(x+4)^2 > 0$

$-5(-5-2)(-5-4)(-5+4)^2 > 0 \Rightarrow -315 > 0$ is False

Testing : Region 2 (x=-1) : $x(x-2)(x-4)(x+4)^2 > 0$

$-1(-1-2)(-1-4)(-1+4)^2 > 0 \Rightarrow -135 > 0$ is False

Testing : Region 3 (x=1) : $x(x-2)(x-4)(x+4)^2 > 0$

$1(1-2)(1-4)(1+4)^2 > 0 \Rightarrow 75 > 0$ is True

Testing : Region 4 (x=3) : $x(x-2)(x-4)(x+4)^2 > 0$

$3(3-2)(3-4)(3+4)^2 > 0 \Rightarrow -147 > 0$ is False

Testing : Region 5 (x=5) : $x(x-2)(x-4)(x+4)^2 > 0$

$5(5-2)(5-4)(5+4)^2 > 0 \Rightarrow 1215 > 0$ is True

Testing : Boundary points $(x = 0\,, 2\; 4\,, -4)$

$$x(x-2)(x-4)(x+4)^2 > 0$$

$x = 0 \Longrightarrow 0(0-2)(0-4)(0+4)^2 > 0 \Longrightarrow 0 > 0$ is False

$x = 2 \Longrightarrow 2(2-2)(2-4)(2+4)^2 > 0 \Longrightarrow 0 > 0$ is False

$x = 4 \Longrightarrow 4(4-2)(4-4)(4+4)^2 > 0 \Longrightarrow 0 > 0$ is False

$x = -4 \Longrightarrow -4(-4-2)(-4-4)(-4+4)^2 > 0 \Longrightarrow 0 > 0$ is False

solution set is : $\{x: x \in R\,, 0 < x < 2 \text{ or } x > 4\} = (0,2) \cup (4,\infty)$

Example 9 :

Use the test point method to solve the polynomial inequality :

A) $x^2 + 6x + 9 \geq 0$, B) $x^2 + 6x + 9 > 0$

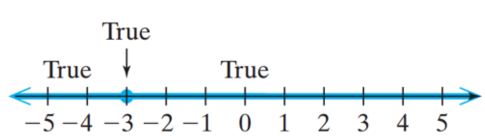
C) $x^2 + 6x + 9 \leq 0$, D) $x^2 + 6x + 9 < 0$

Solution :

$x^2 + 6x + 9 = 0 \Longrightarrow (x+3)^2 = 0 \Longrightarrow x = -3$ is boundary point

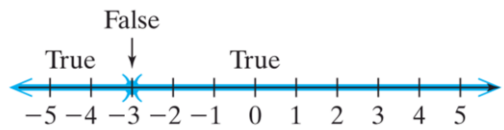
A) $x^2 + 6x + 9 \geq 0$

$$(x+3)^2 \geq 0$$



Solution set is $R = (-\infty, \infty)$

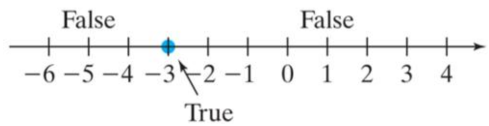
B) $x^2 + 6x + 9 > 0$



$(x + 3)^2 > 0$

Solution set is $R/\{-3\} = (-\infty, -3) \cup (-3, \infty)$

C) $x^2 + 6x + 9 \leq 0$



$(x + 3)^2 \leq 0$

Solution set is $\{-3\}$

D) $x^2 + 6x + 9 < 0$

$(x + 3)^2 < 0$

A perfect square cannot be negative

Solution set is $\emptyset = \{\ \}$

Note :

The test method can be used to solve the rational inequalities , A Rational Inequality is an inequality in which one or more terms is a rational expression · the solution set to rational inequality must exclude all values of the variable that make the inequality undefined , that is exclude all values that make the denominator equal to zero for any rational expression in the inequality .

Example 10 :

Use the test point method to solve the rational inequality :

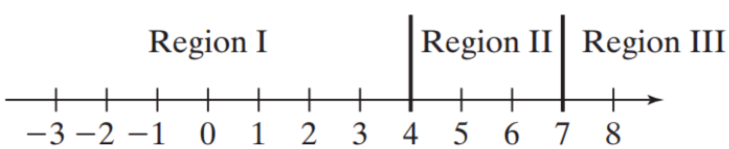
$$\frac{x+2}{x-4} \le 3$$

Solution :

$$\frac{x+2}{x-4} = 3 \Rightarrow x+2 = 3x-12 \Rightarrow 2x = 14 \Rightarrow x = 7$$

$$x-4 = 0 \Rightarrow x = 4$$

Boundary points are : 4 , 7



Testing : Region 1 (x=0) : $\frac{x+2}{x-4} \le 3$

$\frac{0+2}{0-4} \le 3 \Rightarrow \frac{-1}{2} \le 3$ is True

Testing : Region 2 (x=5) : $\frac{x+2}{x-4} \le 3$

$\frac{5+2}{5-4} \le 3 \Rightarrow 7 \le 3$ is False

Testing : Region 3 (x=8) : $\frac{x+2}{x-4} \leq 3$

$\frac{8+2}{8-4} \leq 3 \implies \frac{5}{2} \leq 3$ is True

Testing : boundary points (x=4 , 7) : $\frac{x+2}{x-4} \leq 3$

$x = 4 \implies \frac{4+2}{4-4} \leq 3 \implies \frac{6}{0} \leq 3$ $undefined$ is false

$x = 7 \implies \frac{7+2}{7-4} \leq 3 \implies 3 \leq 3$ is True

The solution set is : $\{x : x \in R , x < 4 \text{ or } x \geq 7\} = (-\infty, 4) \cup [7, \infty)$

Example 11 :

Use the test point method to solve the rational inequality :

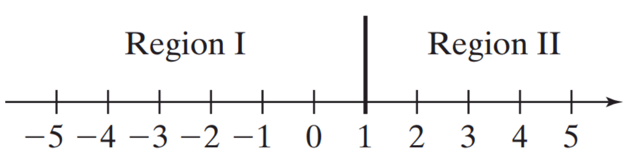
$$\frac{3}{x-1} > 0$$

Solution :

$$\frac{3}{x-1} = 0 \implies 3 = 0 \implies impossible$$

$$x - 1 = 0 \implies x = 1$$

Boundary points is : 1



Testing : Region 1 (x=0) : $\frac{3}{x-1} > 0$

$\frac{3}{0-1} > 0 \Rightarrow -3 > 0$ is False

Testing : Region 2 (x=2) : $\frac{3}{x-1} > 0$

$\frac{3}{2-1} > 0 \Rightarrow 3 > 0$ is True

Testing : boundary points (x=1) : $\frac{3}{x-1} \leq 3$

$x = 1 \Rightarrow \frac{3}{1-1} > 0 \Rightarrow \frac{3}{0} > 0$ $undefined$ is false

The solution set is : $\{x : x \in R\ ,\ x > 1\} = (1, \infty)$

5 - Solving Absolute Value Equation

Absolute Value Equation of the form $|x| = a$

If a is a real number , then :

1) If $a \geq 0$ the equation $|x| = a$ is equivalent to x=a or x=-a
2) If $a < 0$, there is no solution to the equation $|x| = a$

Example 12 :

solve the absolute value equation :

A) $|x| = 5$ B) $|x| = 0$ C) $|x| = -6$ D) $|2w - 3| = 5$

Solution :

A) $|x| = 5 \Rightarrow x = 5 \ \ or \ \ x = -5$

B) $|x| = 0 \Rightarrow x = 0$

C) $|x| = -6 \Rightarrow no\ solution$

D) $|2w - 3| = 5 \Rightarrow 2w - 3 = 5 \ \ or \ \ 2w - 3 = -5$

$2w = 8 \ \ or \ \ 2w = -2$

$w = 4 \ \ or \ \ w = -1$

Solving Equation Have Two Absolute Value

Some equation have two absolute values , the solutions of the equation : $|x| = |y|$ are $x = y$ or $x = -y$. that is if two quantities have the same absolute value then the quantities are equal or the quantities are opposites .

Example 13 :

solve the absolute value equation : $|2w-3| = |5w+1|$

Solution :

$$|2w-3| = |5w+1|$$

Either $2w-3 = 5w+1 \Rightarrow 3w = -4 \Rightarrow w = \frac{-4}{3}$

Or $2w-3 = -5w-1 \Rightarrow 7w = 2 \Rightarrow w = \frac{2}{7}$

Example 14 :

solve the absolute value equation : $|x-4| = |x+8|$

Solution :

$$|x-4| = |x+8|$$

Either $x-4 = x+8 \Rightarrow -4 = 8 \Rightarrow impossible$

Or $x-4 = -x-8 \Rightarrow 2x = -4 \Rightarrow x = -2$

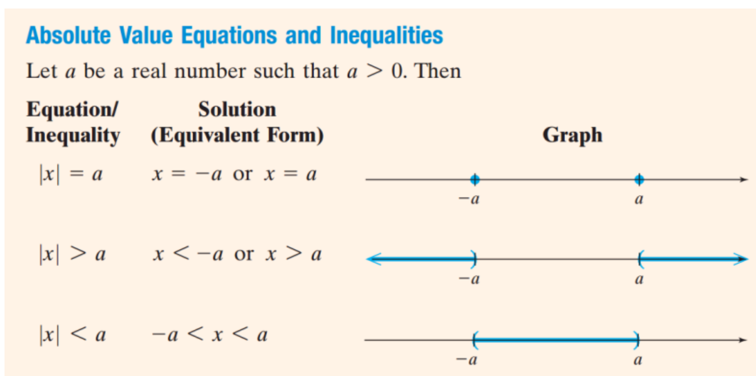
6- Absolute Value Inequalities

Solving Absolute Value Inequalities by Definition

In the previous section we studied absolute value equations in the form $|x| = a$, in this section we will solve absolute value inequalities ·

An inequality in any of the forms $|x| < a$, $|x| \leq a$, $|x| > a$, $|x| \geq a$ is called an absolute value inequality ·

Recall that an absolute value represents distance from zero on the real number line ·



Absolute Value Equations and Inequalities

Let a be a real number such that $a > 0$. Then

Equation/ Inequality	Solution (Equivalent Form)	Graph
$\lvert x\rvert = a$	$x = -a$ or $x = a$	$-a$ a
$\lvert x\rvert > a$	$x < -a$ or $x > a$	$-a$ a
$\lvert x\rvert < a$	$-a < x < a$	$-a$ a

<u>Example 15 :</u>

solve the absolute value Inequalities :

A) $|3w - 1| - 4 < 7$ **B)** $3 \leq 1 + \left|\frac{1}{2}t - 5\right|$

<u>Solution :</u>

A) $|3w + 1| - 4 < 7$

$$|3w + 1| < 11$$

$$-11 < 3w + 1 < 11$$

$$-12 < 3w < 10$$

$$-4 < w < \frac{5}{2}$$

B) $3 \le 1 + \left|\frac{1}{2}t - 5\right|$

$$\left|\frac{1}{2}t - 5\right| \ge 2$$

$$\frac{1}{2}t - 5 \ge 2 \quad or \quad \frac{1}{2}t - 5 \le -2$$

$$\frac{1}{2}t \ge 7 \quad or \quad \frac{1}{2}t \le 3$$

$$t \ge 14 \quad or \quad t \le 6$$

Solving Absolute Value Inequalities By the Test Point Method

The absolute value inequality was converted to an equivalent compound inequality , however sometimes students have difficulty setting up the appropriate compound inequality , to avoid this problem you may want to use the test point method to solve absolute value inequalities .

Summary

- ✓ Find the boundary points of the inequality .
- ✓ Plot the boundary points in the number line , this divides the number line into regions .
- ✓ Select a test point from each regions and substitute it in the original inequality .
- ✓ If the test point makes the original inequality true , then that region is part of the solution set .
- ✓ Test the boundary points in the original inequality .
- ✓ If a boundary point makes the original inequality true then that point is a part of the solution set .

Example 16 :

solve the absolute value Inequalities by the test point method :

A) $|3w-1|-4<7$ B) $3 \le 1+\left|\frac{1}{2}t-5\right|$

Solution :

A) $|3w+1|-4<7$

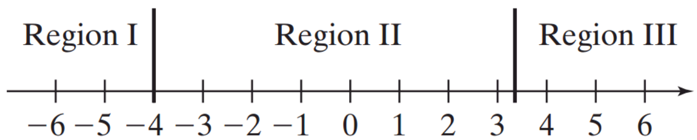
$|3w+1|<11$

$|3w+1|=11$

$3w + 1 = 11 \quad or \quad 3w + 1 = -11$

$3w = 10 \quad or \quad 3w = -12$

$w = \frac{10}{3} \quad or \quad w = -4$ *are boundary points* .



Testing :Region 1 (w=-5)

$|3(-5) + 1| - 4 < 7$

$|-14| - 4 < 7$

$10 < 7$ *is False*

Testing : Region 2 (w=0)

$|3(0) + 1| - 4 < 7$

$|1| - 4 < 7$

$-3 < 7$ *is True*

Testing : Region 3 (w=4)

$|3(4) + 1| - 4 < 7$

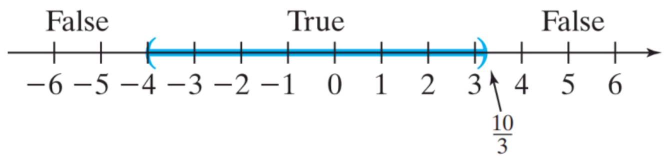
$|13| - 4 < 7$

$9 < 7$ *is False*

Testing (Boundary Points)

$w = \frac{10}{3} \Rightarrow \left|3(\frac{10}{3}) + 1\right| - 4 < 7 \Rightarrow 7 < 7$ is False

$w = -4 \Rightarrow |3(-4) + 1| - 4 < 7 \Rightarrow 7 < 7$ is False



Solution is : $w : w \in R \ , (-4 < w < \frac{10}{3}) = (-4, \frac{10}{3})$

B) $3 \le 1 + \left|\frac{1}{2}t - 5\right|$

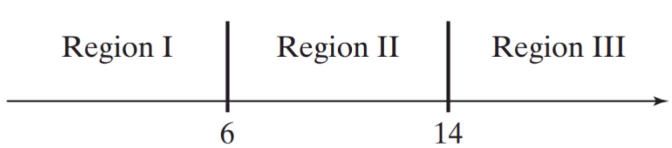
$$1 + \left|\frac{1}{2}t - 5\right| = 3$$

$$\left|\frac{1}{2}t - 5\right| = 2$$

$$\frac{1}{2}t - 5 = 2 \quad or \quad \frac{1}{2}t - 5 = -2$$

$$\frac{1}{2}t = 7 \quad or \quad \frac{1}{2}t = 3$$

$t = 14 \quad or \quad t = 6$ are boundary points



Testing :Region 1 (t=0)

$$3 \le 1 + \left|\frac{1}{2}(0) - 5\right|$$

$3 \le 6$ is True

Testing :Region 2 (t=10)

$$3 \le 1 + \left|\frac{1}{2}(10) - 5\right|$$

$3 \le 1$ is False

Testing :Region 3 (t=16)

$$3 \le 1 + \left|\frac{1}{2}(16) - 5\right|$$

$3 \le 4$ is True

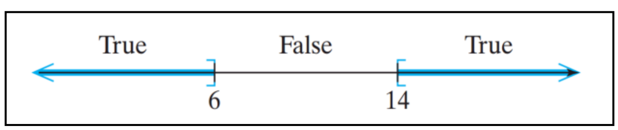
Testing :Boundary points (t=6 , 14)

$$t = 6 \implies 3 \le 1 + \left|\frac{1}{2}(6) - 5\right|$$

$3 \le 3$ is True

$$t = 14 \implies 3 \le 1 + \left|\frac{1}{2}(14) - 5\right|$$

$3 \le 3$ is True



Solution set is : $\{t : t \in R, t \le 6 \text{ or } x \ge 14\} = (-\infty, 6] \cup [14, \infty)$

7-Linear Inequality in two Variables

Graphing Linear Inequalities in two variables

A linear inequality in Two variables x and y is an inequality that can be written in one of the following forms :

$$ax + by < c \,, ax + by \leq c \,,\; ax + by > c \,, ax + by \geq c$$

Provided a and b are not both zero .

A solution to a linear inequality in two variables is an ordered pair that makes the inequality true .

To graph a linear inequality in two variables we will follow this steps.

Summary :

- Solve for y if , if possible .
- Graph the related equation : Draw a dashed line if the inequality is strict , otherwise draw a solid line .
- Shade above or below the line as follow :
- Shade above the line if the inequality is of the form:

  $y > ax + b$ or $y \geq ax + b$
- Shade below the line if the inequality is of the form:

  $y < ax + b$ or $y \leq ax + b$
- Test any point from the region that is shadow .

Example 17 :

Graph a linear inequality in two variables of : $-3x + y \leq 1$

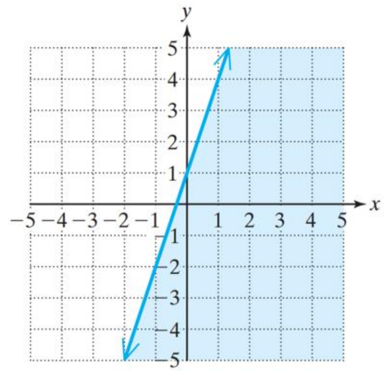
Solution :

$-3x + y \leq 1 \implies y \leq 3x + 1$

Graph : $y = 3x + 1$

$x = 0 \implies y = 1 \implies (0, 1)$

$y = 0 \implies x = -\frac{1}{3} \implies (-\frac{1}{3}, 0)$



Testing : (2,1)

$-3x + y \leq 1 \implies -3(2) + 1 \leq 1 \implies -5 \leq 1$ is True

Example 18 :

Graph a linear inequality in two variables of : $2x + y \geq -4$

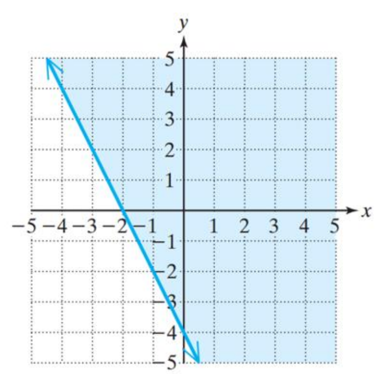
Solution :

$2x + y \geq -4 \implies y \leq -2x - 4$

Graph : $y = -2x - 4$

$x = 0 \implies y = -4 \implies (0, -4)$

$y = 0 \implies x = -2 \implies (-2, 0)$



Testing : (-1,1)

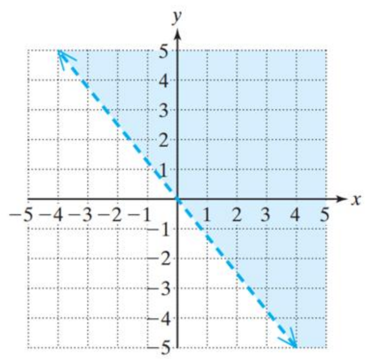
$$2x + y \geq -4 \Rightarrow 2(-1) + 1 \geq -4 \Rightarrow -1 \geq -4 \text{ is True}$$

Example 19 :

Graph a linear inequality in two variables of : $-4y < 5x$

Solution :

$$-4y < 5x \Rightarrow y > -\frac{5}{4}x$$



Graph : $y = -\frac{5}{4}x$

$$x = 0 \Rightarrow y = 0 \Rightarrow (0,0)$$

$$x = -4 \Rightarrow y = 5 \Rightarrow (-4,5)$$

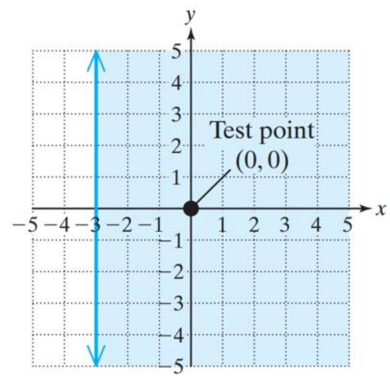
Testing : (1,1)

$$4y < 5x \Rightarrow 4(1) < 5(1) \Rightarrow 4 < 5 \text{ is True}$$

Example 20 :

Graph a linear inequality in two variables of : $4x \geq -12$

Solution :



$4x \geq -12 \Rightarrow x \geq -3$

Graph : $x = -3$ is a vertical line

Testing : (0,0)

$4x \geq -12 \Rightarrow 4(0) < -12 \Rightarrow 0 < -12$ is True

Graphing a Compound Linear Inequalities in two variables

Example 21 :

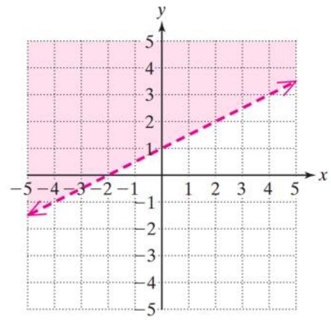
Graph a compound linear inequality in two variables of :

$\mathrm{y} > \frac{1}{2}\mathrm{x} + 1$ and $x + y < 1$

Solution :

1st Inequality

$$y > \frac{1}{2}x + 1 \Rightarrow y = \frac{1}{2}x + 1$$

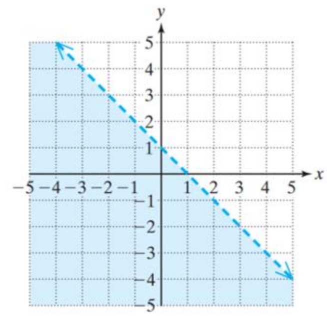


$$x = 0 \Rightarrow y = 1 \Rightarrow (0, 1)$$

$$y = 0 \Rightarrow x = -2 \Rightarrow (-2, 0)$$

2nd Inequality

$$x + y < 1 \Rightarrow y < -x + 1 \Rightarrow y = -x + 1$$

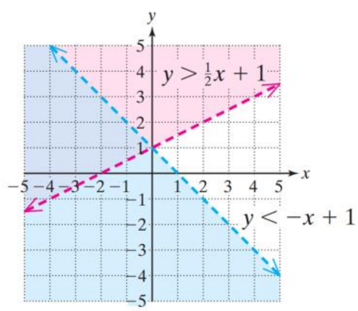


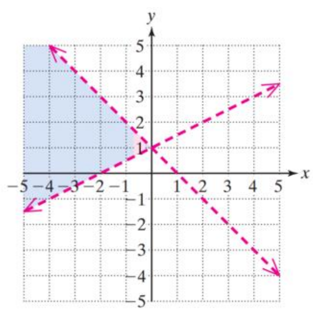
$$x = 0 \Rightarrow y = 1 \Rightarrow (0, 1)$$

$$y = 0 \Rightarrow x = 1 \Rightarrow (1, 0)$$

Intersection

The region bounded by the inequalities is the region above the line $y = \frac{1}{2}x + 1$ *and below the line* $y = -x + 1$ *this is intersection of the two regions as shown in this two figures :*





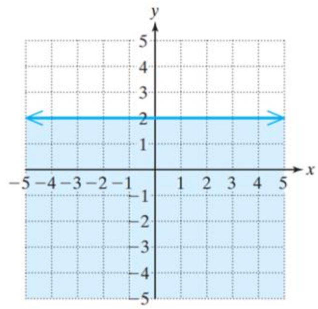
Example 22 :

Graph a compound linear inequality in two variables of :

$3y \le 6$ *or* $y - x \le 0$

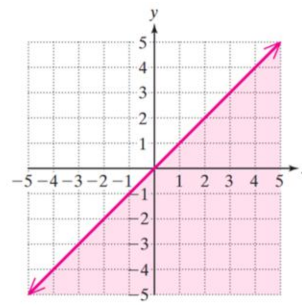
Solution :

1^{st} Inequality



$3y \le 6 \Rightarrow y \le 2 \Rightarrow y = 2$

2^{nd} Inequality



$y - x \le 0 \Rightarrow y \le x \Rightarrow y = x$

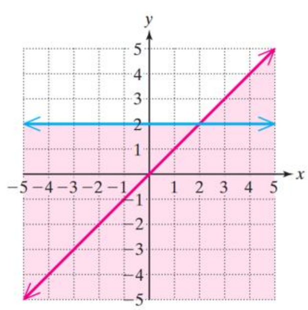
$x = 0 \Rightarrow y = 0 \Rightarrow (0,0)$

$y = 1 \Rightarrow x = 1 \Rightarrow (1,1)$

Union

the solution to the compound inequality $3y \le 6$ *or* $y - x \le 0$

is the union of the two regions as shown in this figure :



Example 23 :

Graph a compound linear inequality in two variables of :

$x \leq 0$ and $y \geq 0$

Solution :

1^{st} Inequality

$\mathbf{x \leq 0}$

On the y axis and the 2^{nd} and 3^{rd} quadrants

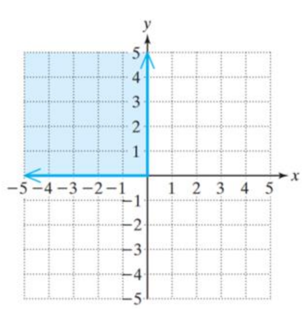
2^{nd} Inequality

$\mathbf{y \geq 0}$

On the x axis and in the 1^{st} and 2^{nd} quadrants .

Intersection

the solution to this inequalities is the intersection of these regions and is the set of all points in the 2^{nd} quadrants with the boundary included as shown in this figure :



CHECK POINT

Example H.W :

1) Find $(-\infty, 2) \cap (-5, \infty)$

2) Find $(-\infty, -3) \cap (-\infty, 0)$

3) Solve the compound inequality : $5x + 2 \geq -8$ and $-4x > -24$

4) Solve the inequality : $-6 < 5 - 2x \leq 1$

5) Solve the compound inequality $-10t - 8 \geq 12$ or $3t - 6 > 3$

6) Solve the inequality : $f(x) = x^2 + 3x - 4$ graphically $>, < 0$

7) Solve the inequality : $f(x) = \frac{1}{x-2} > 0$, $f(x) = \frac{1}{x-2} < 0$ graphically .

By using the Test point method solve the inequalities :

8) $x^2 + x > 6$

9) $x(x-5)(x+2)^2 > 0$

10) A) $x^2 - 4x + 4 \geq 0$

B) $x^2 - 4x + 4 > 0$

C) $x^2 - 4x + 4 \leq 0$

D) $x^2 - 4x + 4 < 0$

11) $\frac{x-5}{x+4} \leq -1$

12) $\frac{-5}{x+2} < 0$

13) solve the absolute value equation : $|4x + 1| = 9$

14) Solve the absolute value equation : $|3-2x| = |3x-1|$

15) Solve the absolute value inequality : $|6x-5| - 4 \leq 3$

16) Solve the absolute value inequality : $|3x+6| - 3 \geq 6$

17) Solve the inequalities by using the test point method for :

A) $10 \geq 6 + |3x-4|$

B) $\left|\frac{1}{2}x + 4\right| + 1 > 6$

18) Graph the solution set of : $-3y < x$

19) Graph the solution set of : $2 \leq -2x$

20) Graph the solution set of : $2x - 3y < 6$

21) Graph the solution set of : $4x - y > -4$

22) Graph the solution of the compound inequalities :

A) $x - 3y > 3$ and $y < -2x + 4$

B) $2y \leq 4$ or $y \leq x + 1$

C) $x \leq 0$ and $y \leq 0$