

4- Gaussian - Elimination with inverse method

Using the following augmented matrix

$$AX = B$$

$$[A | I | B] \Rightarrow [I | A^{-1} | X]$$

using elementary transformation

Ex 2

Use the Gaussian - elimination with
inverse method

$$\begin{aligned}x - 2y + 3z &= 9 \\-x + 3y &= -4 \\2x - 5y + 5z &= 17\end{aligned}$$

Sol.

$$A = \left[\begin{array}{ccc|ccc} 1 & -2 & 3 & 1 & 0 & 0 \\ -1 & 3 & 0 & 0 & 1 & 0 \\ 2 & -5 & 5 & 0 & 0 & 1 \end{array} \right]$$

$R_1 + R_2$; $-2R_1 + R_3$

$$\left[\begin{array}{ccc|ccc} 1 & -2 & 3 & 1 & 0 & 0 \\ 0 & 1 & 3 & 1 & 1 & 0 \\ 0 & -1 & -1 & -2 & 0 & 1 \end{array} \right]$$

$2R_2 + R_1$; $R_2 + R_3$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 9 & 3 & 2 & 0 \\ 0 & 1 & 3 & 1 & 1 & 0 \\ 0 & 0 & 2 & -1 & 1 & 1 \end{array} \right]$$

17

$$+\frac{1}{2}R_3 \quad \left[\begin{array}{ccc|ccc|c} 1 & 0 & 9 & 3 & 2 & 0 & 19 \\ 0 & 1 & 3 & 1 & 1 & 0 & 5 \\ 0 & 0 & 1 & -\frac{1}{2} & +\frac{1}{2} & \frac{1}{2} & 2 \end{array} \right]$$

$$-3R_3 + R_2 \quad ; \quad -9R_3 + R_1$$

$$\left[\begin{array}{ccc|ccc|c} 1 & 0 & 0 & \frac{15}{2} & -\frac{5}{2} & -\frac{9}{2} & 1 \\ 0 & 1 & 0 & \frac{5}{2} & -\frac{1}{2} & -\frac{3}{2} & -1 \\ 0 & 0 & 1 & -\frac{1}{2} & \frac{1}{2} & \frac{1}{2} & 2 \end{array} \right]$$

$$[I \mid A^{-1} \mid X]$$