* Equivalent Matrices : $A \sim B$

If $r(A) = r(B)$, then the matrix (A) is equivalent to the matrix B and is written as follows $A \sim B$.

If $r(A) \neq r(B)$, then the matrix (A) is not equivalent to the matrix (B) and is written as follows $A \nsim B$

— Steps to solve the equivalent matrix

1- Find the rank of the original matrix
2- Find the Canonical form
3- Find the rank of Canonical form
4- A Comparison of $r(A)$ and $r(B)$, if they are equal, then are equivalent, but if they are not equal, they are not equivalent.

Ex If you have

$$A = \begin{bmatrix} 1 & 2 & 3 & -1 \\ 0 & 1 & 0 & 0 \\ 0 & 4 & 0 & 0 \\ -1 & -2 & 0 & 1 \end{bmatrix}$$

Find the equivalence between matrix (A) and its Canonical form (B)

sol: :

1- $|A| = \sum_{j=1}^{4} a_{ij}\alpha_{ij}$

$\alpha_{ij} = (-1)^{i+j} |M_{ij}|$

Select R_2

$= \sum a_{2j}\alpha_{2j}$

$= a_{21}\alpha_{21} + a_{22}\alpha_{22} + a_{23}\alpha_{23} + a_{24}\alpha_{24}$

$= 0 + (1)(-1)^4 |M_{22}| + 0 + 0$

$= (1) \begin{vmatrix} 1 & 3 & -1 \\ 0 & 0 & 0 \\ -1 & 0 & 1 \end{vmatrix} \begin{matrix} 1 & 3 \\ 0 & 0 \\ -1 & 0 \end{matrix}$

$= \{0\} - \{0\} = 0$

delet C_1 and R_2 for matrix(A)

$A^* = \begin{bmatrix} 2 & 3 & -1 \\ 4 & 0 & 0 \\ -2 & 0 & 1 \end{bmatrix}$

$|A^*| = \begin{vmatrix} 2 & 3 & -1 \\ 4 & 0 & 0 \\ -2 & 0 & 1 \end{vmatrix} \begin{matrix} 2 & 3 \\ 4 & 0 \\ -2 & 0 \end{matrix}$

$= [0+0+0] - \{0+0+12\} = -12 \neq 0$

$\therefore r(A) = 3$

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$$A = \begin{bmatrix} 1 & 2 & 3 & -1 \\ 0 & 1 & 0 & 0 \\ 0 & 4 & 0 & 0 \\ -1 & -2 & 0 & 1 \end{bmatrix}$$

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2- $R_1 + R_4$ $\begin{bmatrix} 1 & 2 & 3 & -1 \\ 0 & 1 & 0 & 0 \\ 0 & 4 & 0 & 0 \\ 0 & 0 & 3 & 0 \end{bmatrix}$

$-4R_2 + R_3$ $\begin{bmatrix} 1 & 2 & 3 & -1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 3 & 0 \end{bmatrix}$

$R_4 \Longleftrightarrow R_3$ $\begin{bmatrix} 1 & 2 & 3 & -1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

$\frac{1}{3} R_3$ $\begin{bmatrix} 1 & 2 & 3 & -1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} = B$

3- delete R_4, C_4

$$|B^*| = \begin{vmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} \begin{matrix} 1 & 2 \\ 0 & 1 \\ 0 & 0 \end{matrix}$$

$$[1+0+0] - [0+0+0] = 1 \neq 0$$

$$\therefore r(B) = 3$$

4- $r(A) = 3 = r(B)$

$\therefore A \sim B$ equivalent matrix