* The vector product : الضرب الاتجاهي

The general formula is as follows

$$\vec{U} * \vec{V} = \left(\begin{vmatrix} u_2 & u_3 \\ v_2 & v_3 \end{vmatrix}, - \begin{vmatrix} u_1 & u_3 \\ v_1 & v_3 \end{vmatrix}, \begin{vmatrix} u_1 & u_2 \\ v_1 & v_2 \end{vmatrix} \right)$$

E.g. If $\vec{U} = (0, 1, 3)$, $\vec{V} = (2, -1, 4)$

and $\vec{W} = (3, 0, -1)$

Find

(1-) $\vec{U} * \vec{W}$ (2-) $\vec{U} \cdot \vec{W}$

Sol.:

1- $$\vec{U} * \vec{W} = \left(\begin{vmatrix} u_2 & u_3 \\ w_2 & w_3 \end{vmatrix}, - \begin{vmatrix} u_1 & u_3 \\ w_1 & w_3 \end{vmatrix}, \begin{vmatrix} u_1 & u_2 \\ w_1 & w_2 \end{vmatrix} \right)$$

$$= \left(\begin{vmatrix} 1 & 3 \\ 0 & -1 \end{vmatrix}, - \begin{vmatrix} 0 & 3 \\ 3 & -1 \end{vmatrix}, \begin{vmatrix} 0 & 1 \\ 3 & 0 \end{vmatrix} \right)$$

$$= ([-1-0], -[0-9], [0-3])$$

$$= (-1, 9, -3)$$ الناتج متجه

2- $\vec{U} \cdot \vec{W} = (0, 1, 3) \cdot (3, 0, -1) = (0 + 0 + (-3))$

$= \boxed{-3}$

Scanned by CamScanner

* Vector Norms: or (Length of a vector)

The general formula is as follows:-

$$\|\vec{u}\| = \sqrt{u_1^2 + u_2^2 + \dots + u_n^2}$$

For example: If $\vec{v} = (2, 3, 1)$ and $\vec{u} = (-1, 5, 0)$

Find 1- $\|\vec{u}\|$ 2- $\|\vec{u}\| + \|\vec{v}\|$

3- $\|3\vec{u} - 5\vec{v}\|$

Sol.:

1- $\|\vec{u}\| = \sqrt{u_1^2 + u_2^2 + u_3^2} = \sqrt{(-1)^2 + (5)^2 + (0)^2}$

$= \sqrt{1+25} = \sqrt{26}$

2- $\|\vec{u}\| + \|\vec{v}\| = \sqrt{u_1^2 + u_2^2 + u_3^2} + \sqrt{v_1^2 + v_2^2 + v_3^2}$

$= \sqrt{(-1)^2 + (5)^2 + 0^2} + \sqrt{(2)^2 + (3)^2 + (1)^2}$

$= \sqrt{26} + \sqrt{14}$

$= 8.83$

3- $\|3\vec{u} - 5\vec{v}\|$

$3\vec{u} - 5\vec{v} = 3(-1, 5, 0) - 5(2, 3, 1)$

$= (-3, 15, 0) - (10, 15, 5)$

$= (-13, 0, -5)$

$\|(-13, 0, -5\| = \sqrt{(-13)^2 + (0)^2 + (-5)^2}$

$= \sqrt{169 + 25}$

$= \sqrt{194}$

* Distance :-

The distance between vectors $\vec{u}$ and $\vec{v}$ is defined as

$$d(\vec{u},\vec{v}) = \|\vec{u}-\vec{v}\|$$

$$= \sqrt{(u_1-v_1)^2+(u_2-v_2)^2+\dots+(u_n-v_n)^2}$$

E.g If $\vec{u}=(5,3)$, $\vec{v}=(0,-1)$ and $\vec{w}=(2,1)$

Find 1- The distance between vectors $\vec{u}$ and $\vec{v}$
2- $d(\vec{w},\vec{u}) = d(\vec{u},\vec{w})$

Sol.:

1- $d(\vec{u},\vec{v}) = \|\vec{u}-\vec{v}\| = \sqrt{(u_1-v_1)^2+(u_2-v_2)^2}$

$$= \sqrt{(5-0)^2+(3-(-1))^2}$$

$$= \sqrt{25+16} = \sqrt{41}$$

2- $d(\vec{w},\vec{u}) \stackrel{?}{=} d(\vec{u},\vec{w})$

$$\sqrt{(w_1-u_1)^2+(w_2-u_2)^2} = \sqrt{(u_1-w_1)^2+(u_2-w_2)^2}$$

$$\sqrt{(2-5)^2+(1-3)^2} = \sqrt{(5-2)^2+(3-1)^2}$$

$$\sqrt{(-3)^2+(-2)^2} = \sqrt{(3)^2+(2)^2}$$

$$\sqrt{13} = \sqrt{13}$$

* Eigenvalues and eigenvectors :-

ⓐ (2x2) matrix :

The method with a (2x2) matrix (A), Suppose

$$A=\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}_{2\times 2} ; \quad V=\begin{bmatrix} V_1 \\ V_2 \end{bmatrix}_{2\times 1}$$

then consider the eigenvalue equation :-

$$AV = \lambda V$$

$$AV - \lambda V = 0$$

$$(A-\lambda I)V = 0 \quad \cdots (1)$$

$$\left(\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\right)\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\left(\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix}\right)\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad \cdots (2)$$

To find the eigenvalues from $|A-\lambda I| = 0$

$$\underbrace{\begin{bmatrix} a_{11}-\lambda & a_{12} \\ a_{21} & a_{22}-\lambda \end{bmatrix}}_{(A-\lambda I)}\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad \cdots (3)$$

$$|A-\lambda I| = 0 \implies \begin{vmatrix} a_{11}-\lambda & a_{12} \\ a_{21} & a_{22}-\lambda \end{vmatrix} = 0$$

To find the eigenvectors from eq (3), when subsituted the value λ into eq (3)

E.g. Find the eigenvalues and eigenvectors of the matrix. (or Find λ and V) (or Find V)

$$A = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$$

Sol. :

$$(A-\lambda I)V = 0 \quad \cdots (1)$$

$$\left(\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix}\right)\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\left(\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix}\right)\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 2-\lambda & 1 \\ 1 & 2-\lambda \end{bmatrix}\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad \ldots 2$$

To find the values (λ) as follows:

$|A - \lambda I| = 0$

$$\begin{vmatrix} 2-\lambda & 1 \\ 1 & 2-\lambda \end{vmatrix} = 0$$

$(2-\lambda)(2-\lambda) - 1 = 0$

$4 - 2\lambda - 2\lambda + \lambda^2 - 1 = 0$

$\lambda^2 - 4\lambda + 3 = 0$

$(\lambda - 3)(\lambda - 1) = 0$

$\therefore \lambda_1 = 1 \quad ; \quad \lambda_2 = 3$

To find the values (V) as follows:-

either $\lambda_1 = 1$ subsituted in eq(2)

$$\begin{bmatrix} 2-1 & 1 \\ 1 & 2-1 \end{bmatrix}\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$V_1 + V_2 = 0 \quad \ldots 3$

$V_1 + V_2 = 0 \quad \ldots 4$

from eq(3) $V_1 = -V_2$ $\quad \therefore V = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$

or $\lambda_2 = 3$ substituted in eq(2)

$$\begin{bmatrix} 2-3 & 1 \\ 1 & 2-3 \end{bmatrix}\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \Rightarrow \begin{aligned} -V_1 + V_2 &= 0 \quad \ldots 5 \\ V_1 - V_2 &= 0 \quad \ldots 6 \end{aligned}$$

from eq(5) $-V_1 = -V_2 \Rightarrow V_1 = V_2$

$\therefore V = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$