* Methods for Solving equations in case of $m=n$

1- Solve a system of linear equations using matrices (factorization method) or (general method)

$$a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n = b_1$$
$$a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n = b_1$$
$$\vdots$$
$$a_{m1}x_1 + a_{m2}x_2 + \dots + a_{mn}x_n = b_m$$

$$\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & & a_{2n} \\ \vdots & & & \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{bmatrix}$$

or write as follows

$$AX = B$$

and depeding on the following formula:

$$X = A^{-1} \cdot B$$

Ex. If you have

$$x_1 + x_2 = 3$$
$$2x_1 + x_2 = 6$$

Use the matrix method to solve the system of equations.

sol:

$$X = A^{-1} \cdot B$$

$$A = \begin{bmatrix} 1 & 1 \\ 2 & 1 \end{bmatrix}; \quad B = \begin{bmatrix} 3 \\ 6 \end{bmatrix}$$

$$A^{-1} = \frac{adj(A)}{|A|}$$

$$|A| = 1 - 2 = -1 \neq 0$$

$$adj(A) = (Cof(A))^T$$

$$Cof(A) = \begin{bmatrix} \alpha_{11} & \alpha_{12} \\ \alpha_{21} & \alpha_{22} \end{bmatrix}_{2\times 2}$$

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مصفوفة المعاملات نفس رتبة المصفوفة (A)

$$\alpha_{ij} = (-1)^{i+j} |M_{ij}|$$

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نابع المحدد الصغرى (minor) من حذف الصف (i) وحذف العمود (j)

$$\alpha_{11} = (-1)^{1+1} |M_{11}| = (-1)^2 |1| = 1$$

ناتجة من حذف الصف الاول والعمود الاول من المصفوفة (A)

$$\alpha_{12} = (-1)^{1+2} |M_{12}| = (-1)^3 |2| = -2$$

حذف الصف الاول و العمود الثاني

$$\alpha_{21} = (-1)^{2+1} |M_{21}| = (-1)^3 |1| = -1$$

حذف الصف الثاني والعمود الاول

$$\alpha_{22} = (-1)^{2+2} |M_{22}| = (-1)^4 |1| = 1$$

حذف الصف الثاني و العمود الثاني

$$Cof(A) = \begin{bmatrix} 1 & -2 \\ -1 & 1 \end{bmatrix} \quad ; \quad adj(A) = (Cof(A))^T$$

$$adj(A) = \begin{bmatrix} 1 & -1 \\ -2 & 1 \end{bmatrix}$$

$$A^{-1} = \frac{1}{-1} \begin{bmatrix} 1 & -1 \\ -2 & 1 \end{bmatrix}$$

$$X = A^{-1} \cdot B = -1 \begin{bmatrix} 1 & -1 \\ -2 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 6 \end{bmatrix} = (-1) \begin{bmatrix} -3 \\ 0 \end{bmatrix}$$

$$\therefore X = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 3 \\ 0 \end{bmatrix}$$