

* Complex numbers !.

A number which can be written in the form $a+bi$, where a, b are real numbers and $i = \sqrt{-1}$ is called a complex number.

If $z = a+bi$ is the complex number, then a and b are called real and imaginary parts; then

$$z = \underbrace{a}_{\text{Real number}} + \underbrace{bi}_{\text{imaginary number}}$$

Ex

$$z = 2 + 3i$$

$$a = 2$$

$$b = 3$$

$$z = -3 + 6i$$

$$a = -3$$

$$b = 6$$

$$z = -i$$

$$a = 0$$

$$b = -1$$

$$z = 5$$

$$a = 5$$

$$b = 0$$

$$z = -3 - 6i$$

$$a = -3$$

$$b = -6$$

* Operation of complex numbers

① Addition of complex numbers :-

Let $z_1 = a+bi$ and $z_2 = c+di$ be two complex numbers, then

$$\begin{aligned} z_1 + z_2 &= (a+bi) + (c+di) \\ &= \underbrace{(a+c)}_{\text{Real}} + \underbrace{(bi+di)}_{\text{imaginary}} \end{aligned}$$

3- Multiplying Complex Numbers

Let $z_1 = a+bi$ and $z_2 = c+di$, be two complex numbers. Then

$$\begin{aligned} z_1 \cdot z_2 &= (a+bi)(c+di) \\ &= a(c+di) + bi(c+di) \\ &= ac + adi + bci + bdi^2 \\ &= ac + adi + bci - bd \\ &= \underbrace{(ac - bd)}_{\text{real}} + \underbrace{(ad + bc)}_{\text{imaginary}}i \end{aligned}$$

Remark:-

1- $i = \sqrt{-1}$

2- $i^2 = \sqrt{-1} \cdot \sqrt{-1} = -1$

3- $i^3 = i^2 \cdot i = -1 \cdot i = -i$

4- $i^4 = (i^2)^2 = (-1)^2 = 1$

5- $i^5 = i^3 \cdot i^2 = (-i)(-1) = +i$
or $(i^4)^1 \cdot i = 1(i) = i$

Ex Find Multiply the following complex numbers.

1- $(2+8i)(3+7i)$
 $= 2(3+7i) + 8i(3+7i)$
 $= 6 + 14i + 24i + 56i^2$
 $= 6 + 38i - 56 = -50 + 38i$