

* Properties of the inverse :-

1- $A \cdot A^{-1} = A^{-1} \cdot A = I_m$

2- $(A^{-1})^{-1} = A$

3- $(A^T)^{-1} = (A^{-1})^T$

4- $|A^{-1}| = \frac{1}{|A|}$

5- If A is a ^{diagonal} matrix and $|A| \neq 0$; then the inverse of a matrix is diagonal matrix ~~about~~ the elements of a principal diagonal ~~this~~ $\therefore$ the matrix are

$$A = \begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix}; A^{-1} = \begin{bmatrix} \frac{1}{a} & 0 & 0 \\ 0 & \frac{1}{b} & 0 \\ 0 & 0 & \frac{1}{c} \end{bmatrix}$$

6- If A and B are two matrices then

$$(A \cdot B)^{-1} = B^{-1} \cdot A^{-1} \neq A^{-1} \cdot B^{-1}$$

E.g.:

Let $A = \begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix}; B = \begin{bmatrix} 1 & 3 \\ 2 & 5 \end{bmatrix}$

$$A^{-1} = \frac{\text{adj}(A)}{|A|} = \frac{1}{2} \begin{bmatrix} 1 & 6 \\ -1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{2} & 3 \\ -\frac{1}{2} & 1 \end{bmatrix}$$

$$B^{-1} = \frac{\text{adj}(B)}{|B|} = \frac{\begin{bmatrix} 5 & -3 \\ -2 & 1 \end{bmatrix}}{-1}$$

$$= \begin{bmatrix} -5 & 3 \\ 2 & -1 \end{bmatrix}$$

$$B^{-1} \cdot A^{-1} = \begin{bmatrix} -5 & 3 \\ 2 & -1 \end{bmatrix} \cdot \begin{bmatrix} \frac{1}{2} & 3 \\ -\frac{1}{2} & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -4 & 3 \\ \frac{3}{2} & -1 \end{bmatrix}$$

$$A^{-1} \cdot B^{-1} = \begin{bmatrix} \frac{1}{2} & 3 \\ -\frac{1}{2} & 1 \end{bmatrix} \cdot \begin{bmatrix} -5 & 3 \\ 2 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} -\frac{5}{2} & \frac{3}{2} \\ \frac{3}{2} & -\frac{5}{2} \end{bmatrix}$$

$$A \cdot B = \begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 3 \\ 2 & 5 \end{bmatrix} = \begin{bmatrix} 2 & 6 \\ 3 & 8 \end{bmatrix}$$

$$\text{adj}(A \cdot B) = \begin{bmatrix} 8 & -6 \\ -3 & 2 \end{bmatrix}$$

$$(A \cdot B)^{-1} = -\frac{1}{2} \begin{bmatrix} 8 & -6 \\ -3 & 2 \end{bmatrix} = \begin{bmatrix} -4 & 3 \\ \frac{3}{2} & -1 \end{bmatrix} = B^{-1} \cdot A^{-1} = A^{-1} \cdot B^{-1}$$