Euler's Algorithm

Input: a , b , n , y_0, f(x,y)

Step(1): Set x_0=a and compute $h = \frac{b-a}{n}$

Step(2): For k=1,2,…,n-1

Step(3): Evaluate $y_{k+1} = y_k + h\, f(x_k, y_k)$

and $x_{k+1} = x_k + h$

Step(4): print x_k and y_k then stop.

2- **Modified Euler's Method (Heun's Method)**

To solve the I.V.P. $y' = f(x, y)$ with $y(x_0) = y_0$ by this method we use the formula:

$$x_{k+1} = x_k + h$$

$$y_{k+1} = y_k + \frac{h}{2}\left[f(x_k, y_k) + f(x_{k+1}, y_{k+1})\right] \quad \text{for} \quad k = 0, 1, \dots, n-1 \qquad (4)$$

where $y_{k+1} = y_k + h\, f(x_k, y_k)$

This method is predictor – corrector method , and the local truncation error

of modified Euler's method is $O(h^3)$ (L.T.E.= $y''(c_1)\,\frac{-h^3}{12}$)

Example (3): Solve the I.V.P.

$$\frac{dy(x)}{dx} = x - y, \quad y(0) = 1$$

By using modified Euler's method at the points x=0.1 , 0.2 , 0.3 , 0.4

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Solution:

$f(x,y)=x-y$, $x_0=0$, $y_0=1$, $h=0.1$, $n=4$

$y_{k+1} = y_k + \frac{h}{2}[f(x_k , y_k)+f(x_{k+1} , y_{k+1})]$ for $k = 0, 1,2,3$

$k=0 \longrightarrow$ $y(x_1)=y(0.1)=y_1= y_0 + \frac{h}{2}[f(x_0 , y_0)+f(x_1 , y_1)]$

$y_1= y_0 + h\, f(x_0 , y_0)=1+(0.1)(x_0-y_0)=1+(0.1)(0-1)=0.9$

$y(x_1)=1+ \frac{0.1}{2}[f(0 , 1)+f(0.1 , 0.9)]= 1+ \frac{0.1}{2}[1+(0.1- 0.9)]=0.91$

$k=1 \longrightarrow$ $y(x_2)=y(0.2)=y_2= y_1+ \frac{h}{2}[f(x_1 , y_1)+f(x_2 , y_2)]$

$y_2= y_1+ h\, f(x_1 , y_1)= 0.9+(0.1)(0.1-0.9)=0.82$

$y(x_2)= 0.91+ \frac{0.1}{2}[f(0.1 , 0.91)+f(0.2 , 0.82)]=0.8385$

$k=2 \longrightarrow$ $y(x_3)=y(0.3)=y_3= y_2+ \frac{h}{2}[f(x_2 , y_2)+f(x_3 , y_3)]$

$=0.782435$

$k=3 \longrightarrow$ $y(x_4)=y(0.4)=y_4= 0.741604$

Example (4): Use modified Euler's method to approximate the solution for the initial value problem

$$y' = x^2 + 4x - \frac{1}{2}y \quad , \quad y(0) = 4 \quad , \quad 0 \le x \le 0.25$$

with h=0.05.

Solution: H.W.

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Modified Euler's Algorithm (Heun's Algorithm)

Input: a , b , n , y_0, f(x,y)

Step(1): Set x_0=a and compute $h = \frac{b-a}{n}$

Step(2): For k=0,1,2,…,n-1

Step(3): Set $x_{k+1} = x_k + h$

Step(4): Find $k_1 = h\, f(x_k , y_k)$

$K_2 = h\, f(x_{k+1} , y_k + k_1)$

Step(5): Evaluate $y_{k+1} = y_k + \frac{1}{2}(k_1 + k_2)$

Step(6): Print x_{k+1} and y_{k+1} then stop.

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