احصاء حيوي

الكورس الاول

(موضوع المحاضرة)

تحليل التباين بمعيار واحد

(مجموع مربعات الانحرافات المشتركة تساوي صفر)

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$$\sum_{i=1}^{k}\sum_{j=1}^{n}\left(Y_{ij}-\bar{Y}_{i}.\right)\left(\bar{Y}_{i}.-\bar{Y}..\right)=\text{Zero}$$

برهان :

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$$\sum_{i=1}^{k}\sum_{j=1}^{n}\left(Y_{ij}-\bar{Y}_{i}.\right)\left(\bar{Y}_{i}.-\bar{Y}..\right)$$

$$=\sum_{i=1}^{k}\sum_{j=1}^{n}\left(Y_{ij}\times\bar{Y}_{i}.-\bar{Y}_{i}.\times\bar{Y}_{i}.-Y_{ij}\times\bar{Y}..+\bar{Y}_{i}.\times\bar{Y}..\right)$$

$$\therefore\ \bar{Y}_{i}.=\frac{Y_{i}.}{n}=\frac{\sum_{j=1}^{n}Y_{ij}}{n}\quad;\quad\bar{Y}..=\frac{Y..}{kn}=\frac{\sum_{i=1}^{k}\sum_{j=1}^{n}Y_{ij}}{kn}=\frac{\sum_{i=1}^{k}Y_{i}.}{kn}$$

$$=\sum_{i=1}^{k}\sum_{j=1}^{n}\left(Y_{ij}\times\frac{Y_{i}.}{n}-\frac{Y_{i}^{2}.}{n^{2}}-Y_{ij}\times\frac{Y..}{kn}+\frac{Y_{i}.}{n}\times\frac{Y..}{kn}\right)$$

برهان :

$$=\sum_{i=1}^{k}\left(\frac{Y_{i}\cdot}{n}\right)\sum_{j=1}^{n}\left(Y_{ij}\right)-n\sum_{i=1}^{k}\left(\frac{Y_{i}^{2}\cdot}{n^{2}}\right)-\frac{Y_{..}}{kn}\sum_{i=1}^{k}\sum_{j=1}^{n}\left(Y_{ij}\right)+\frac{Y_{..}}{kn}\,n\sum_{i=1}^{k}\left(\frac{Y_{i}\cdot}{n}\right)$$

$$=\sum_{i=1}^{k}\left(\frac{Y_{i}\cdot}{n}\right)\times Y_{i}\cdot-n\sum_{i=1}^{k}\left(\frac{Y_{i}^{2}\cdot}{n^{2}}\right)-\frac{Y_{..}}{kn}\times Y_{..}+\frac{Y_{..}}{kn}\times\frac{n}{n}\times\sum_{i=1}^{k}\left(Y_{i}\cdot\right)$$

$$=\cancel{\sum_{i=1}^{k}\left(\frac{Y_{i}^{2}\cdot}{n}\right)}-\cancel{\sum_{i=1}^{k}\left(\frac{Y_{i}^{2}\cdot}{n}\right)}-\cancel{\frac{Y_{..}^{2}}{kn}}+\cancel{\frac{Y_{..}^{2}}{kn}}=\text{Zero}$$

$$\therefore\sum_{i=1}^{k}\sum_{j=1}^{n}\left(Y_{ij}-\bar{Y}_{i}\cdot\right)\left(\bar{Y}_{i}\cdot-\bar{Y}_{..}\right)=\text{Zero}$$