5.2: Continuous probability distribution:

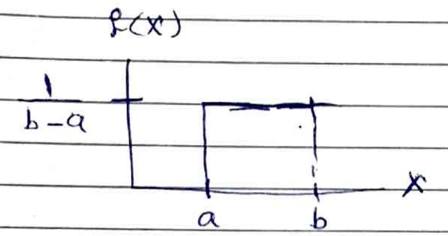
1. Uniform distribution

A very simple dist. for a continuous r.v. is the uniform dist.

A r.v. X is said to have uniform distribution in the interval $[a,b]$ if its p.d.f. is given by:

$$f(x) = \begin{cases} \frac{1}{b-a} & a < x < b \\ 0 & o.w. \end{cases} \qquad X \sim U(a,b)$$

where a and b are real constants with $a < b$ the graph of $f(x)$ is shown in figer below:



4. The distribution function $(F(x))$ is given by

$$F(x) = \begin{cases} 0 & x < a \\ \frac{x-a}{b-a} & a \le x < b \\ 1 & x \geq b \end{cases}$$

Ex:

Buses arrive at a bus stop at 10-minute intervals starting at 6 A.M., That is they arrive at 6, 6:10, 6:20, 6:30 and so on. If a passenger arrives at the bus stop at a time that is uniformly distributed between

6 and 6:20, find the probability that he waits for:

1) Less than 7 minutes for a bus?
2) More than 4 minutes for a bus?

Sol.:
Let x denote the number of minutes past 6 that the passenger arrives at the bus stop, then

$$f(x) = \begin{cases} \frac{1}{20} & 0 < x < 20 \\ 0 & o.w. \end{cases}$$

1) The Passanger will wait for less than 7 minutes if he arrives between 6:3 and 6:10 or between 6:13 at 6:20. Hence the required probability is

$$P(3 < x < 10) + P(13 < x < 20) = \int_3^{10} \frac{1}{20}\, dx + \int_{13}^{20} \frac{1}{20}\, dx$$

$$= \frac{7}{10}$$

2) The passenger will wait for more than 4 minutes if he arrives between 6. and 6:4 or between 6:10 and 6:14, so the required probability is

$$P(0 < x < 4) + P(10 < x < 14) = \int_0^{4} \frac{1}{20}\, dx + \int_{10}^{14} \frac{1}{20}\, dx$$

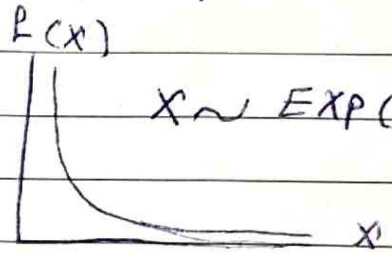
$$= \frac{2}{5}$$

② Exponential distribution: التوزيع الأسي

A r.v. is said to have an exponential dist. if its P.d.f. is given by:

$$f(x) = \begin{cases} \frac{1}{\theta} e^{-x/\theta} & , x \geq 0 \quad , \theta > 0 \\ 0 & o.w \end{cases}$$

$X \sim EXP(\theta)$



or $f(x) = \begin{cases} \theta e^{-\theta x} & , x \geq 0 \\ 0 & o.w \end{cases}$

* The distribution function $F(x)$ is given by:

$$F(x) = \int_0^x f(u)\,du = \int_0^x \frac{1}{\theta} e^{-u/\theta}\,du = 1 - e^{-x/\theta} \quad , x > 0$$

EX:

The milage (in thousand of miles) which car owners get with certain kind of tyre is a r.v. having an exponential with $\theta = 20$, Find the probability that one of these tyres will last

1) At most 12000 miles
2) Anywhere from 18000 To 24000 miles.

Sol.:

Let x denoted the milage of this tyre (in 1000 miles) Then

$$P(X \leq 12) = \int_0^{12} \frac{1}{20} e^{-x/20}\,dx = -e^{-\frac{x}{20}}\Big]_0^{12}$$

$$= 1 - e^{-\frac{12}{20}} = 1 - 0.548 = 0.452$$