Lecture -14-

$$P(18 < X < 24) = \int_{18}^{24} \frac{1}{20} e^{-x/20} \, dx$$

$$= e^{\frac{-18}{20}} - e^{\frac{-24}{20}}$$

$$= 0.406 - 0.301$$

$$= 0.105$$

③ Gamma distribution:

The gamma distribution is an important generalization of the exponential distribution.

A r.v. X is said to have a gamma dist. with Parameters α and β, if its P.d.f. is given by:

$$f(x) = \begin{cases} \frac{1}{\beta^{\alpha} \Gamma(\alpha)} x^{\alpha-1} e^{-x/\beta} & , x > 0 \\ 0 & \text{o.w.} \end{cases}$$

where α, β are positive constants and the gamma function $\Gamma(\alpha)$ is defined as:

⊛ $$\Gamma(\alpha) = \int_{0}^{\infty} x^{\alpha-1} e^{-x} \, dx \quad \text{for } \alpha > 0$$

$$= (\alpha-1)\Gamma(\alpha-1) ,$$

⊛ $\Gamma(\frac{1}{2}) = \sqrt{\pi}$

if α is a positive integer, then

$\Gamma(\alpha) = (\alpha-1)!$

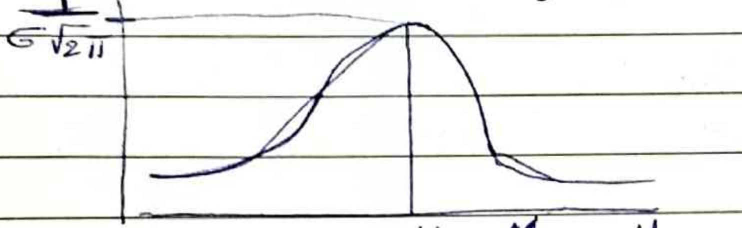
⊛ The P.d.f. of Gamma dist. is reduced to Expo. dist. fun. if $\alpha = 1$, $\beta = \frac{1}{\theta}$

(4) Normal distribution:

The r.v. X is said to have a normal dist. with Parameters μ and σ^2 if its P.d.f. is given by

$$f(x) = \begin{cases} \dfrac{1}{\sqrt{2\pi}\,\sigma} \, e^{-\frac{1}{2\sigma^2}(x-\mu)^2} & -\infty < x < \infty \\ 0 & \text{o.w} \end{cases}$$

where μ and σ are real constants and $\sigma > 0$. The values μ and σ^2 represent, the average value (the mean) and the Possible variation of x (variance). respectively. The graph of $f(x)$ is bell-shape curve, that the symmetric about μ



$\mu = M_o = M_e$

متوسط منوال وسيط

A r.v. X having a normal distribution with Parameters μ and σ^2 by $X \sim N(\mu, \sigma^2)$

Notes:

(1) If $X \sim N(\mu, \sigma^2)$ then $Z = \dfrac{X-\mu}{\sigma}$ is standard normal variate with mean $\mu = 0$ and variance $\sigma^2 = 1$

(2) The P.d.f. of the standard normal Z is

$$f(z) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}z^2} \qquad -\infty < z < \infty$$

and the corrisponding distribution function

$\phi(z) = P(Z \le z)$ is given by

$$\phi(z) = \int_{-\infty}^{z} f(u)\,du = \int_{-\infty}^{z} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}u^2}\,du$$