1.3 Errors in Calculations

Let x^*, y^* are approximation values to the exact values x , y respectively, with the absolute errors e_x , e_y and the relative errors δ_x , δ_y

1- The addition

$$e_{x+y} = |(x+y) - (x^* + y^*)| = e_x + e_y$$

$$\delta_{x+y} = \frac{e_{x+y}}{|x+y|} = \frac{e_x + e_y}{|x+y|} = \frac{|x|\delta_x + |y|\delta_y}{|x+y|}$$

2- The subtraction

$$e_{x-y} = |(x-y) - (x^* - y^*)| = e_x - e_y$$

$$\delta_{x-y} = \frac{e_{x-y}}{|x-y|} = \frac{e_x - e_y}{|x-y|} = \frac{|x|\delta_x - |y|\delta_y}{|x-y|}$$

3- The multiplication

$$e_{xy} = |(xy) - (x^* y^*)|$$

$$= xy - (x - e_x)(y - e_y)$$

$$= xy - \left(xy - ye_x - xe_y + e_x e_y\right)$$

$$= ye_x + xe_y - e_x e_y$$

$$= ye_x + xe_y$$

$$\delta_{xy} = \frac{e_{xy}}{|xy|} = \frac{ye_x + xe_y}{xy}$$

$$= \frac{ye_x}{xy} + \frac{xe_y}{xy}$$

$$= \delta_x + \delta_y$$

4- The division (**Home work)**

$$e_{x/y} = \frac{x}{y}\left(\frac{e_x}{x} - \frac{e_y}{y}\right)$$

$$\delta_{x/y} = \frac{e_{x/y}}{|x/y\,|} = \delta_x - \delta_y$$

<u>Example(2)</u>: Find the upper limit of absolute error in **addition** x+y =12.23+3.12

<u>Solution</u>:

$e_x = \frac{1}{2} \times 10^{-n}$, n=2

$e_x = \frac{1}{2} \times 10^{-2} = \frac{1}{200} = 0.005$

Also

$e_y = \frac{1}{2} \times 10^{-2} = \frac{1}{200} = 0.005$

Then

$e_{x+y} = e_x + e_y = 0.005 + 0.005 = 0.01$

Exercise:

Find the upper limit of absolute error in multiplication x.y =(2.23).(12.3)

Chapter Two

Solutions of Equations in one Variable

Definition: If $f(p)=0$ then p is a root of the equation $f(x)=0$.

Note: If p is the exact root of the equation $f(x)=0$ then p_n is the approximation root and $|p-p_n| \le \varepsilon$ or $|f(p_n)| \le \gamma$ where ε *and* γ are small values accuracy of the solution.

2.1 Determination of roots positions

To locate the position of roots of the function (equation) $f(x)=0$, let $f(x)$ be continuous function and the roots are real

1- $f(x)$ is polynomial

If the value p is a root of the function

$$f(x) = x^n + a_1 x^{n-1} + a_2 x^{n-2} + \cdots + a_n = 0$$

Then $|p| \le 1 + \max\ |a_i| \le 1 + \lambda$ and $\lambda = \max_{\forall i} |a_i|$

__Example(1):__ Find the approximate location of the root for the function

$$f(x)=x^3-2x^2+3x+1=0$$

__Solution:__ By using $|p| \le 1+\lambda$ and $\lambda = \max_{\forall i}|a_i|$ we have

$-(1+\lambda) \le p \le 1+\lambda$,

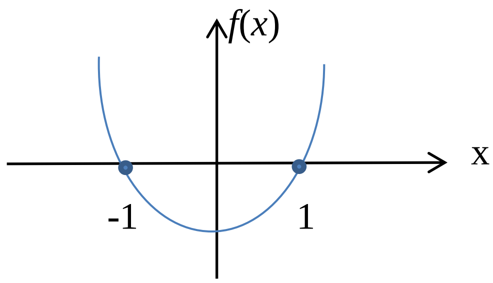
$\lambda = \max_{\forall i}|a_i| = \max\{|1|, |-2|, |3|, |1|\} = 3$

$-4 \le p \le 4$

2- Set the location of the roots using the graph

a- When drawing $y=f(x)$, the points of intersection of the function curve with the x-axis are the roots of the equation $f(x)=0$.

Example(2): Let $f(x)=x^2-1=0$

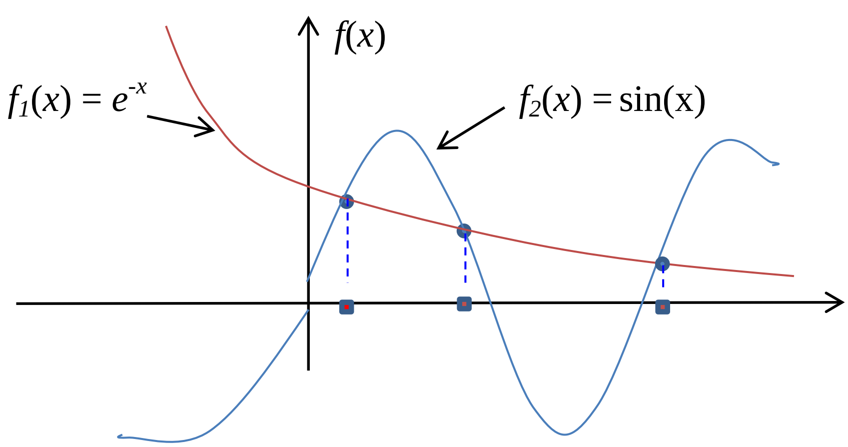


b- If it is difficult to draw the function $y=f(x)$, then we write $f(x)$ as the formula $f_1(x)=f_2(x)$ where $f_1(x)$, $f_2(x)$ everyone can draw easily so that the points of the intersection of the two curves on the axis are the roots of the equation $f(x)=0$.

Example(3): Let $f(x)=e^x\sin(x)-1=0$

we write $f_1(x)=f_2(x)$, i.e.:

$e^x\sin(x)-1=0 \longrightarrow e^x\sin(x)=1 \longrightarrow e^{-x}=\sin(x)$



3- Set roots locations by changing the function signal

let $f(x)$ be continuous function on the interval [a,b], we divide the interval [a,b] into n subintervals $a = x_0 < x_1 < \ldots < x_{n-1} < x_n = b$ where

$x_i = a + ih$, i= 0, 1, … ,n; $h = \frac{b-a}{n}$.

If $f(x_i) \times f(x_{i+1}) < 0$ for any $0 \le i \le n$, then there exits c, a<c<b for which $f(c)=0$.

<u>Example(4)</u>: Find the approximate location of the roots for the functions:

1. $f(x)=x^4-7x^3+3x^2+26x-10=0$ on the interval [-8,8] with n=4 and n=8..
2. $f(x)=x^3+4x^2-10=0$ on the interval [1,2] with n=5.

<u>Solution:</u>

1- Let **n=4**, $h=\frac{b-a}{n}=\frac{8-(-8)}{4}=4$

X	-8	-4	0	4	8
$f(x)$	+	+	-	-	+

There is a root between (-4,0) and (4,8).

If **n=8**, h=2:

X	-8	-6	-4	-2	0	2	4	6	8
f(x)	+	+	+	+	-	+	-	+	+

There is a root between (-2,0), (0,2), (2,4) and (4,6).

2- Let **n=5**, h=0.2

X	1	1.2	1.4	1.6	1.8	2
f(x)	-	-	+	+	+	+

There is a root between (1.2,1.4).